

# Semiconductor Devices and Models

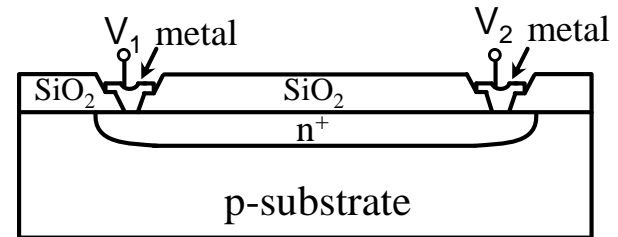
# Semiconductor Devices and Models

- Resistor
- Capacitor
- Diode
- Bipolar Transistor
- MOSFET
- SPICE Model
- Appendix

# Resistors

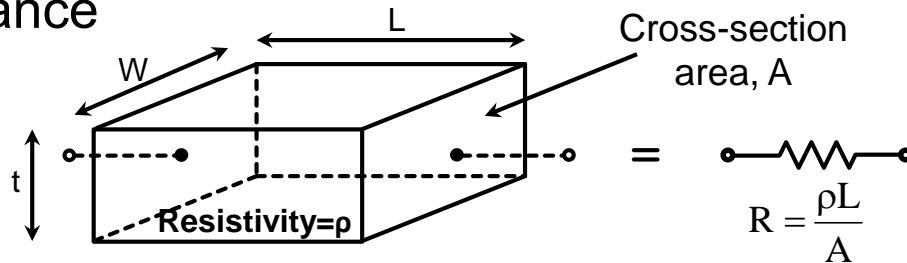
- Material

- ◆ Diffusion layers: e.g. n+, p+, well
- ◆ Conductors: e.g. polysilicon, ...

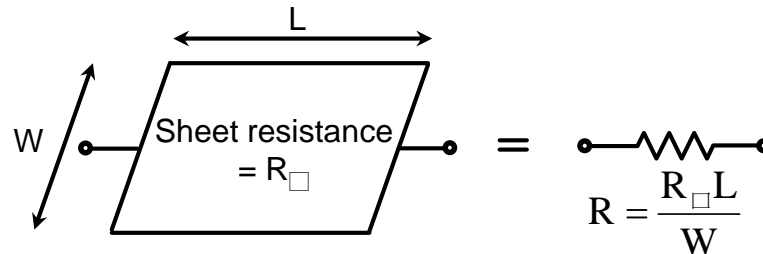


- Resistance calculation

- ◆  $R = \rho L/A = \rho L/tW = R_{\square} L/W$   
 $\rho$  is resistivity  
 $R_{\square} = \rho/t$  is sheet resistance

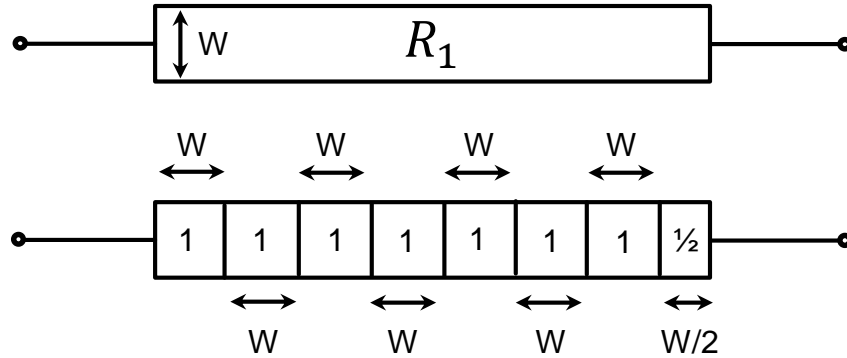


- Sheet resistance ( $R_{\square}$ )

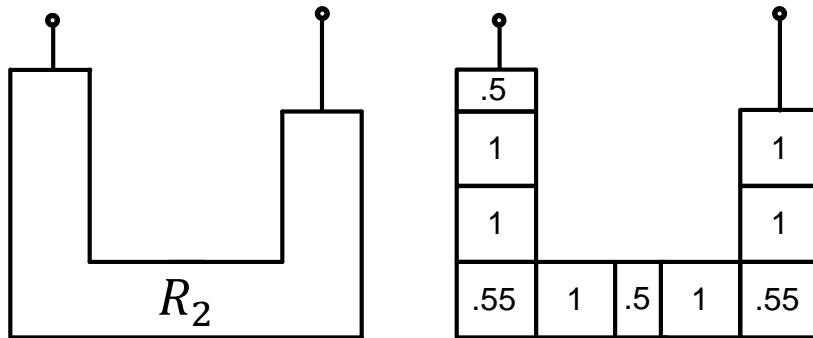


# Resistors (Cont.)

- Graphical calculations from sheet resistance



$$R_1 = 7.5R_{\square}$$

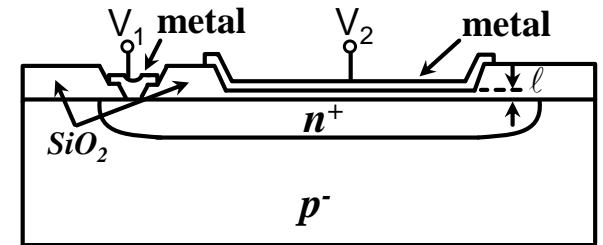


$$R_2 = 8.1R_{\square}$$

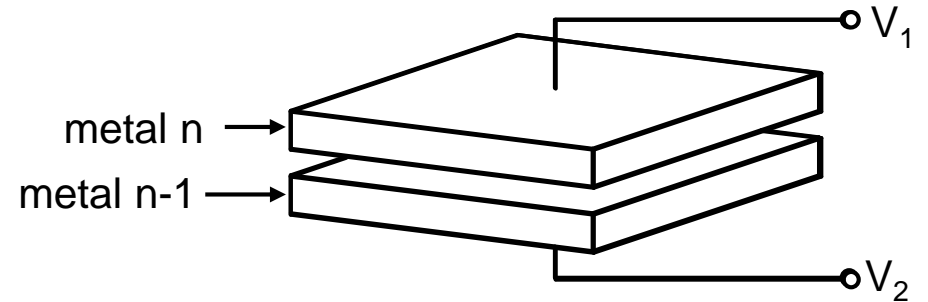
- VC (voltage coefficient) and TC (temperature coefficient) of R (or C)  
 → Nonlinearity → THD

# Capacitors

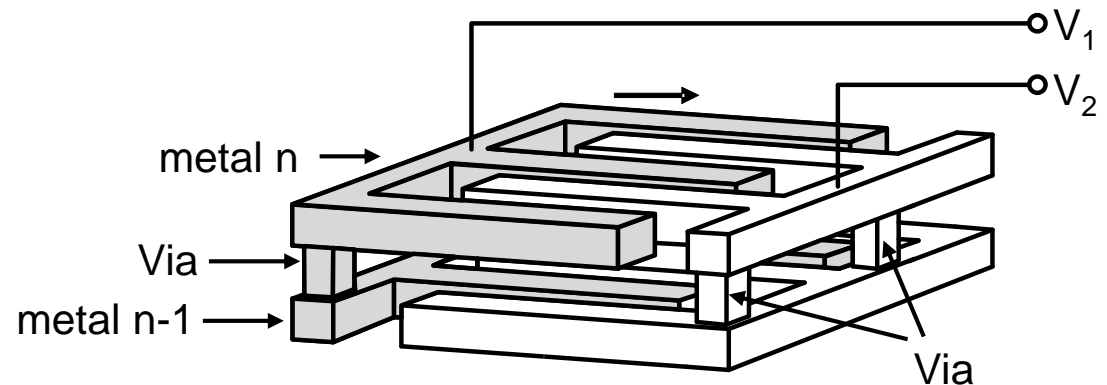
- Metal- or polysilicon- over-diffusion
  - ◆ Capacitance is voltage dependent



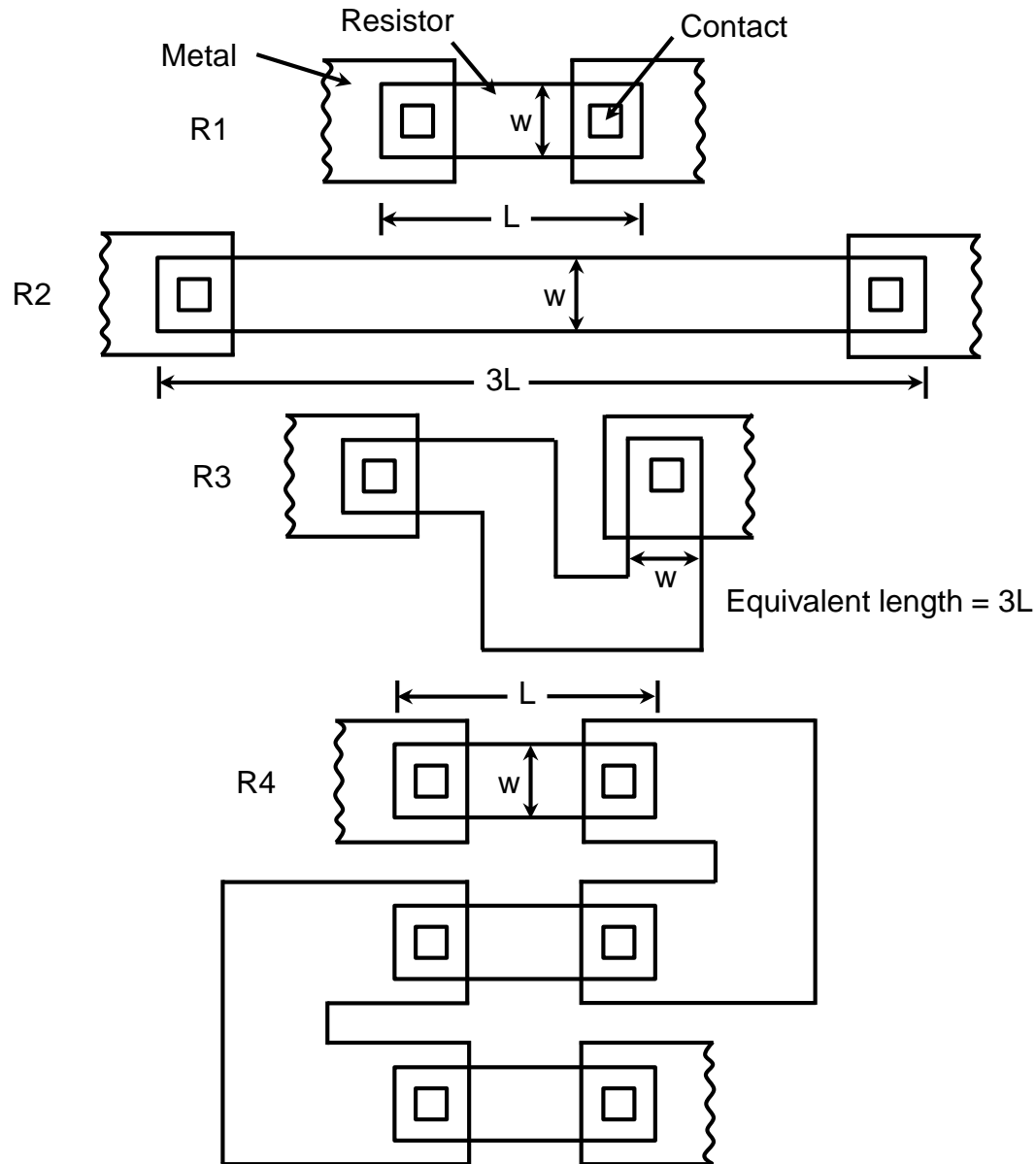
- Metal-Insulator-metal (MIM)
  - ◆ High linearity



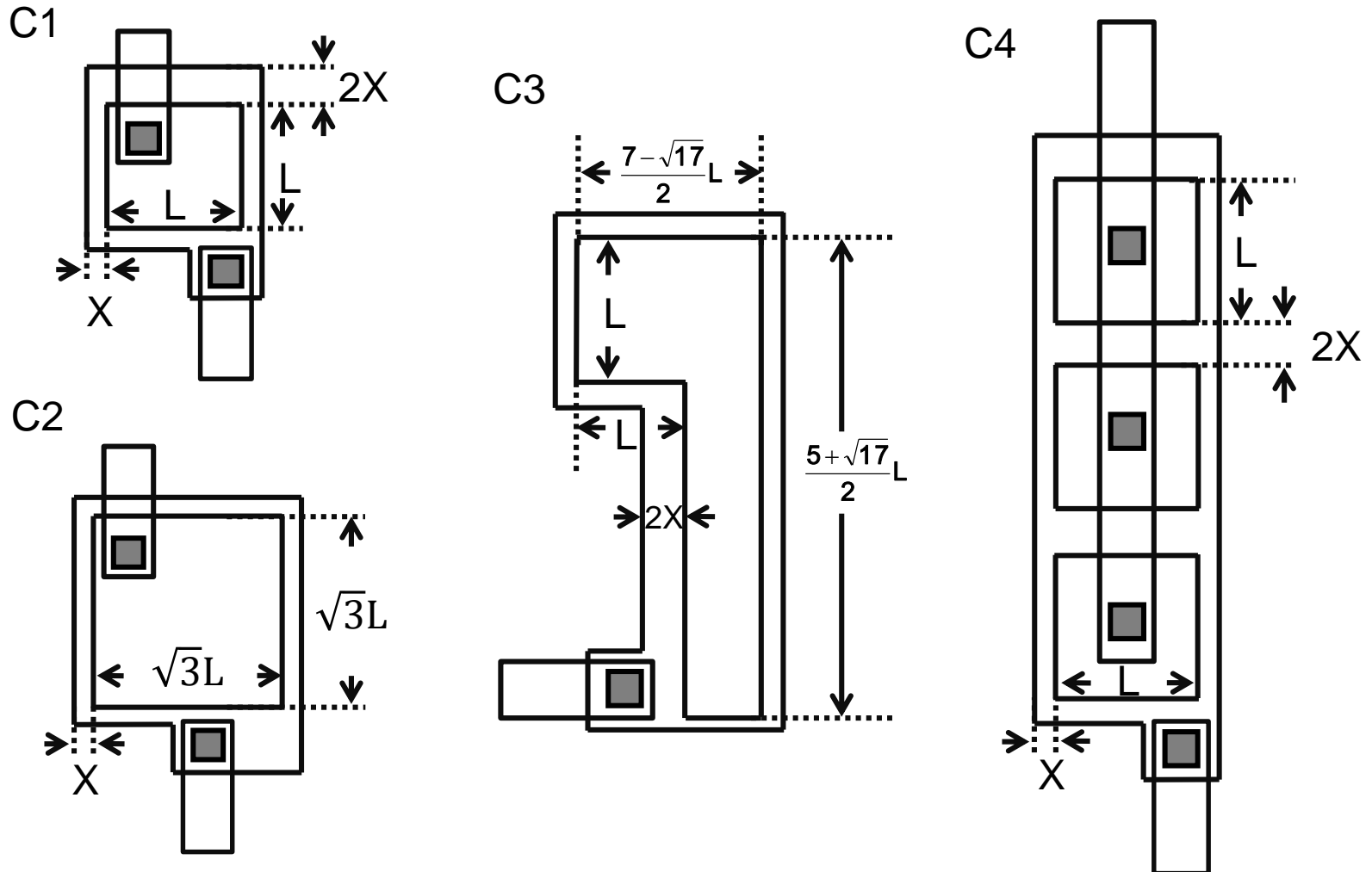
- Inter-metal and intra-metal
  - ◆ Inter: Different layer
  - ◆ Intra: Same layer



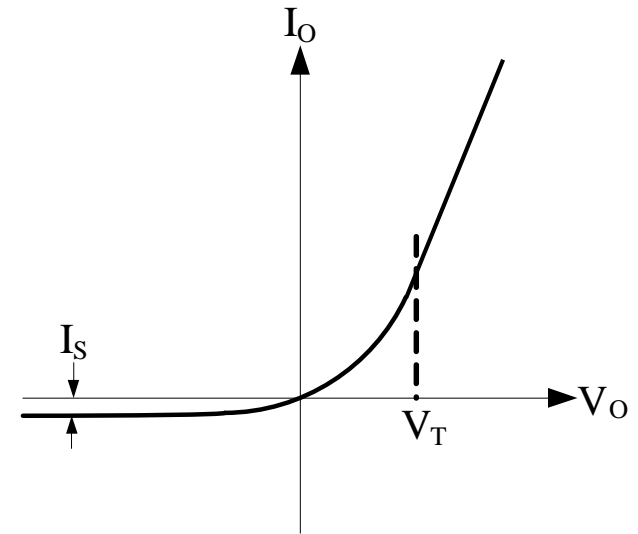
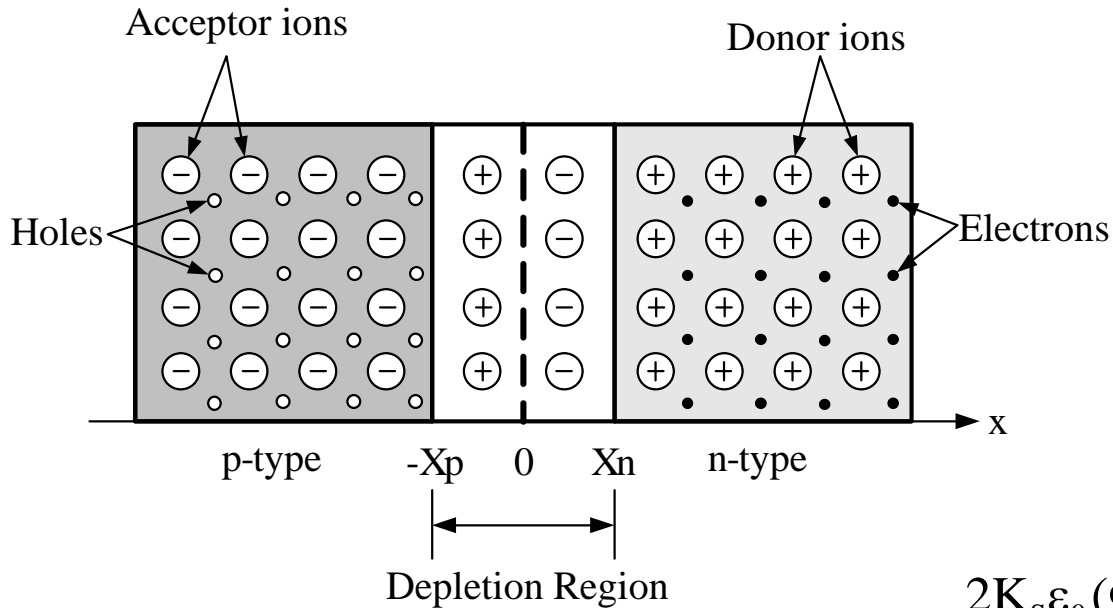
# Resistor Ratio-matching Considerations



# Capacitor Ratio-matching Considerations



# Diode



$$I = I_S \left( \exp^{\frac{v}{V_T}} - 1 \right)$$

where  $V_T = \frac{kT}{q}$

$I_S$  = saturation current

$$X_{p} = \left[ \frac{2K_s \epsilon_0 (\Phi_0 + V_R)}{q} \frac{N_D}{N_A (N_A + N_D)} \right]^{1/2}$$

$$X_{n} = \left[ \frac{2K_s \epsilon_0 (\Phi_0 + V_R)}{q} \frac{N_A}{N_D (N_A + N_D)} \right]^{1/2}$$

$K_s$  : relative permittivity of silicon

$\epsilon_0$  : vacuum permittivity

$\Phi_0$  : built-in potential

$V_R$  : reverse-bias voltage



# Junction Capacitance

- For abrupt junction:

$$Q^+ = Q^- = [2qK_S\epsilon_0(\Phi_0 + V_R)\frac{N_A N_D}{N_A + N_D}]^{1/2}$$

$$C_j = \frac{dQ^+}{dV_R} = \left[ \frac{qK_S\epsilon_0}{2(\Phi_0 + V_R)} \frac{N_A N_D}{N_A + N_D} \right]^{1/2} = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{\Phi_0}}}, \text{ where } C_{j0} = \left[ \frac{qK_S\epsilon_0}{2\Phi_0} \frac{N_A N_D}{N_A + N_D} \right]^{1/2}$$

- For graded junction

$$Q^+ = Q^- = [2qK_S\epsilon_0(\Phi_0 + V_R)\frac{N_A N_D}{(N_A + N_D)}]^{1-m_j}$$

$$C_j = \frac{dQ^+}{dV_R} = (1 - m_j)[2qK_S\epsilon_0\frac{N_A N_D}{N_A + N_D}]^{1-m_j} \frac{1}{(\Phi_0 + V_R)^{m_j}} = \frac{C_{j0}}{(1 + \frac{V_R}{\Phi_0})^{m_j}}$$

$$\text{where } C_{j0} = (1 - m_j)[2qK_S\epsilon_0\frac{N_A N_D}{N_A + N_D}]^{1-m_j} \frac{1}{\Phi_0^{m_j}}$$

$m_j$  depends on the doping profile

$m_j \approx 1/3$  for a linearly graded junction

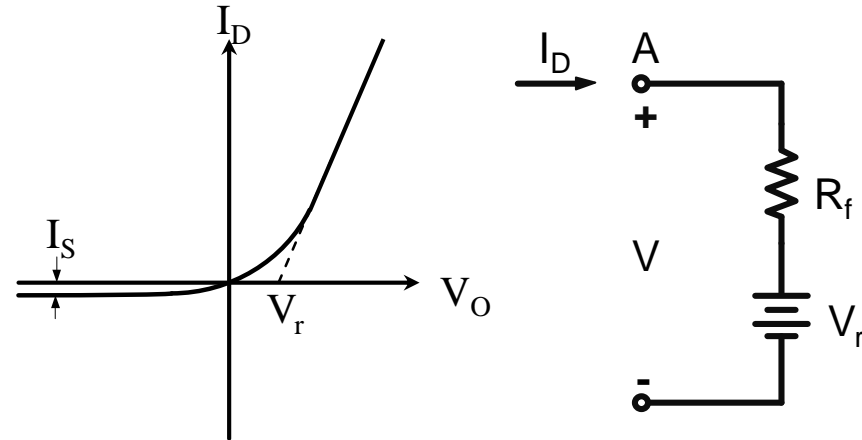
# Diode Model

- DC Model

- ◆ For  $V < V_r$  (off)

- ◆ For  $V > V_r$  (on)

$$R_f = \left(\frac{dI}{dV}\right)^{-1} = \frac{V_T}{I_S e^{V_D/V_T}} \approx \frac{V_T}{I_D}$$



- Small Signal Model

- ◆ For forward-biased diode

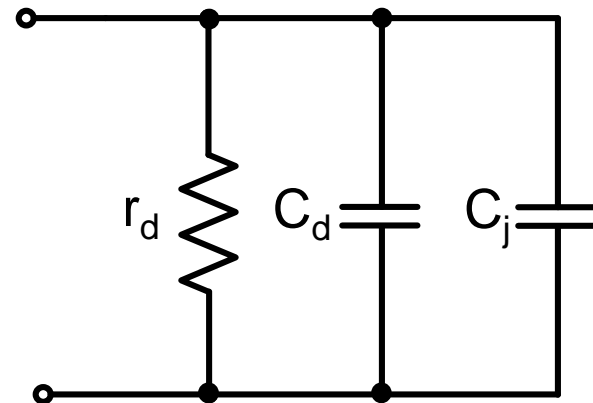
$$r_d \approx \frac{V_T}{I_D}$$

Diffusion capacitance:  $C_d \approx \frac{\tau_T}{r_d}$

Junction capacitance:  $C_j$

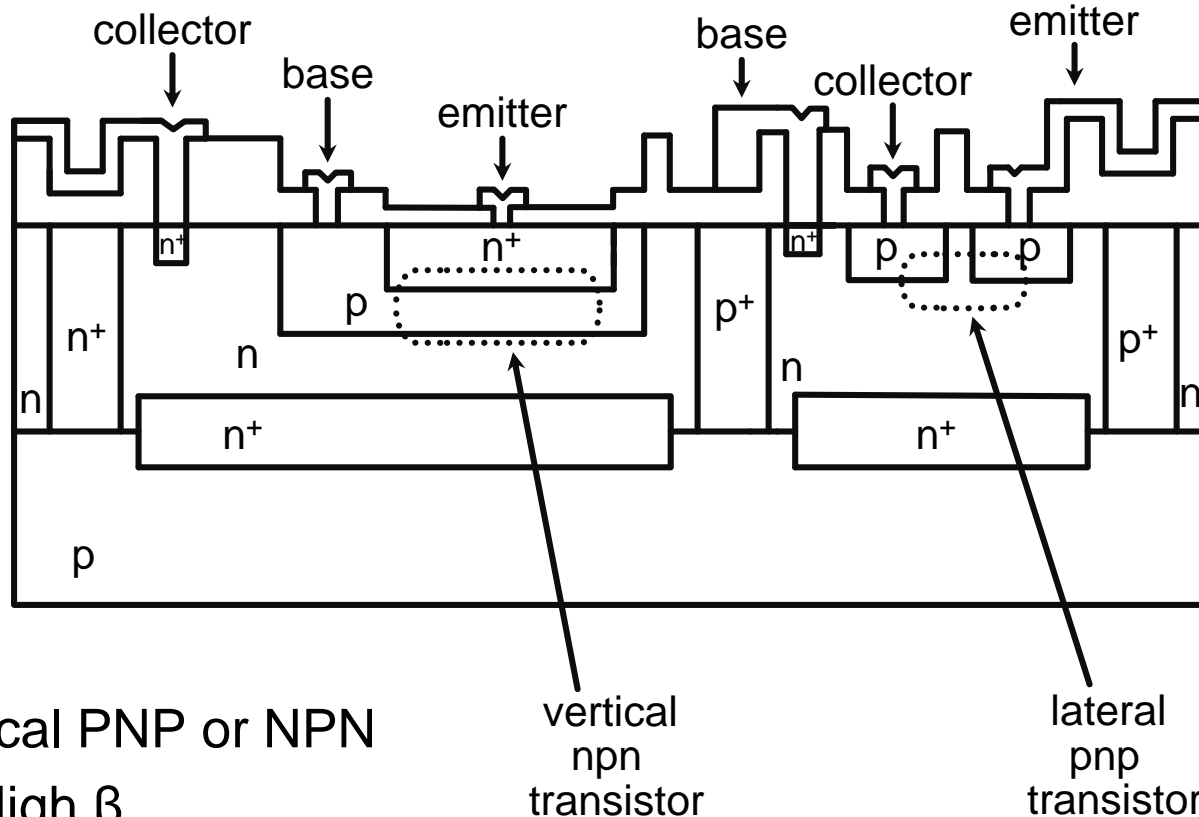
Normally,  $C_d \gg C_j$  ( $C_d \approx 100C_j$ )

$\tau_T$  : Diode transit time



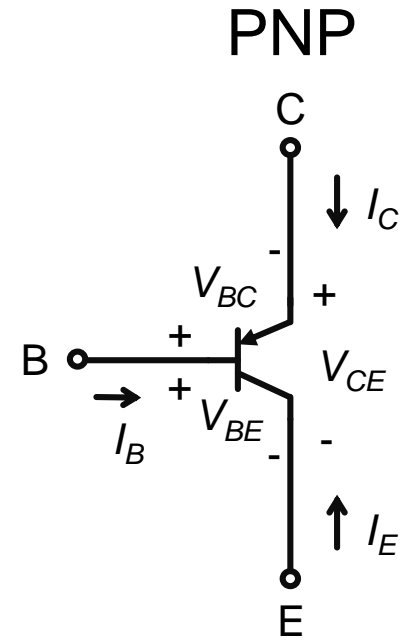
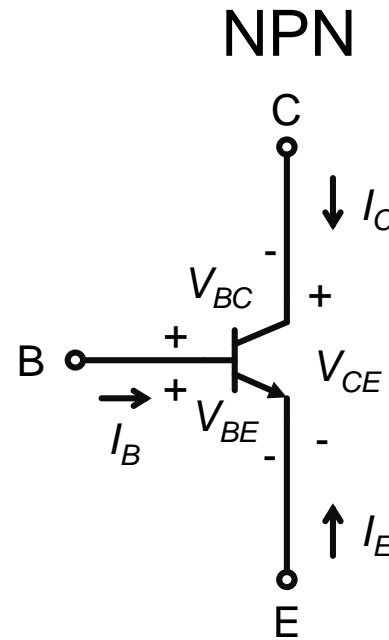
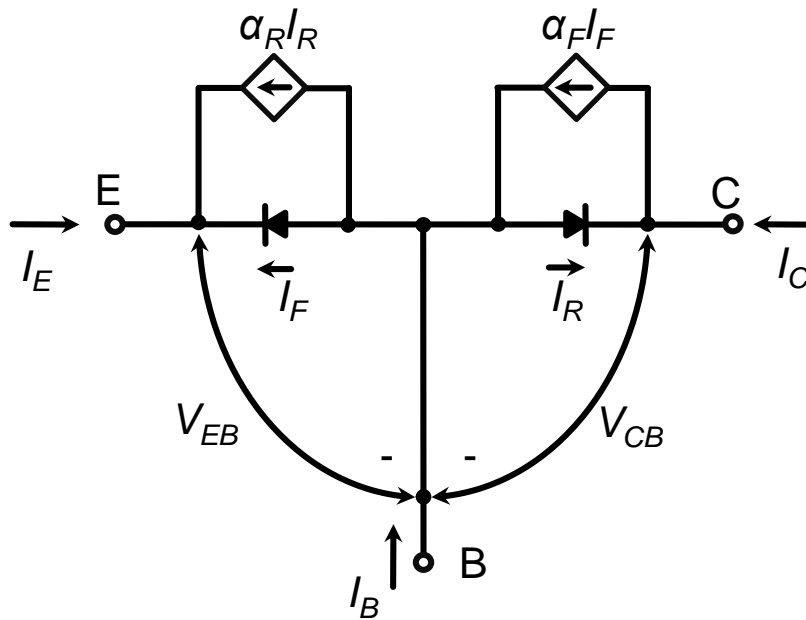
# Bipolar Process

- Vertical and lateral transistor in a bipolar process



- ◆ Vertical PNP or NPN
  - High  $\beta$
- ◆ Lateral PNP or NPN
  - Low  $\beta$

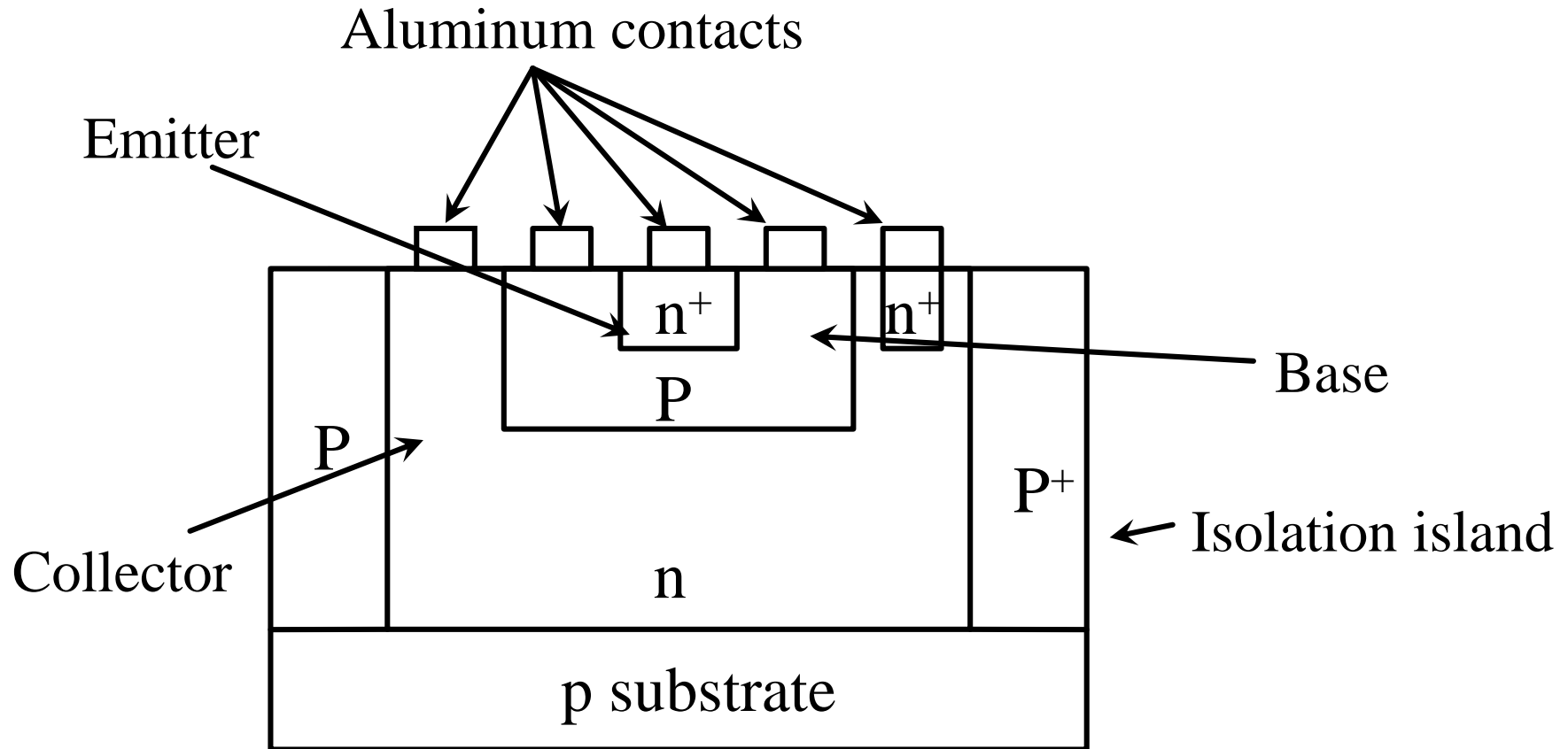
# BJT Model: Ebers-Moll model (DC Model)



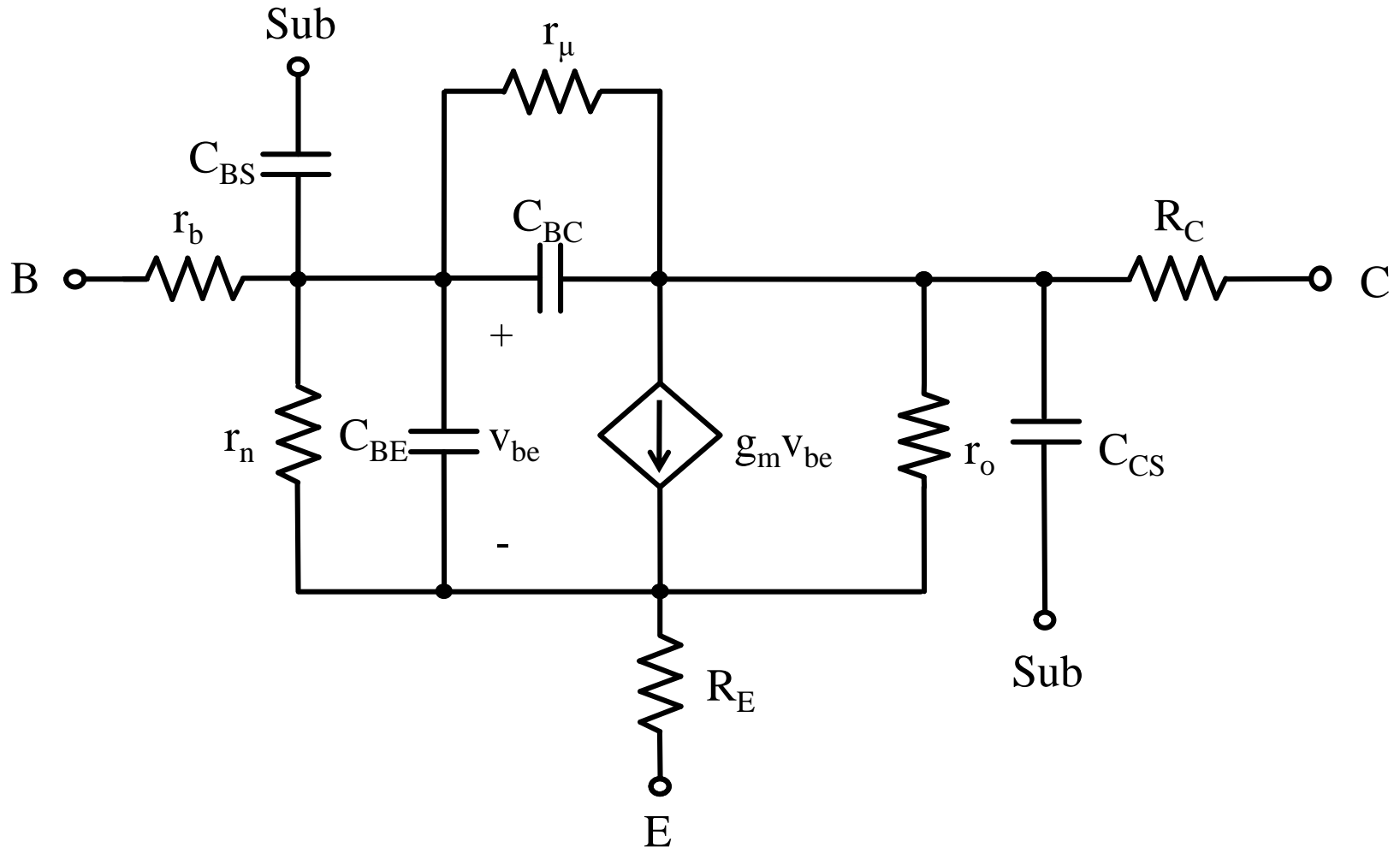
$$I_C = I_S \left[ e^{\left( \frac{V_{BE}}{V_T} \right)} - 1 \right] - \frac{I_S}{\alpha_R} \left[ e^{\left( \frac{V_{BC}}{V_T} \right)} - 1 \right]$$

$$I_E = \frac{-I_S}{\alpha_R} \left[ e^{\left( \frac{V_{BE}}{V_T} \right)} - 1 \right] - I_S \left[ e^{\left( \frac{V_{BC}}{V_T} \right)} - 1 \right]$$

# Small Signal BJT Model



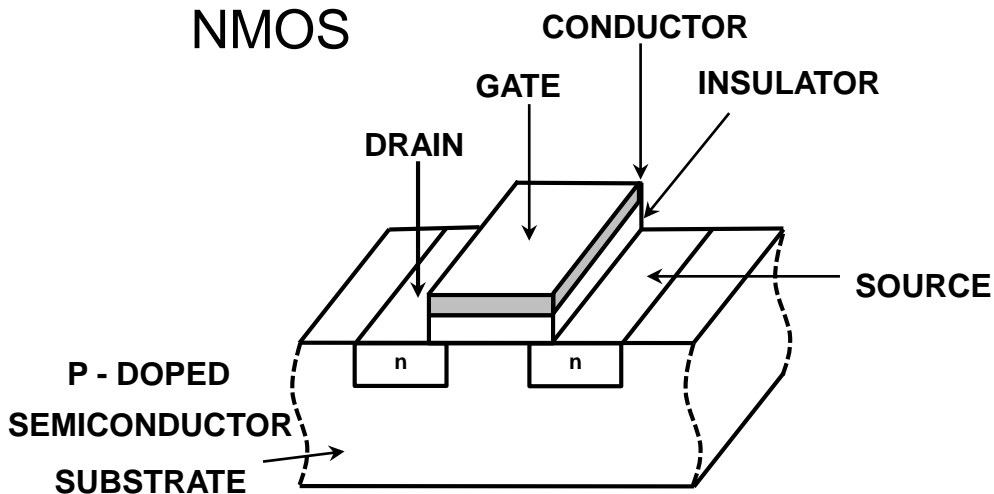
# Small Signal BJT Model (Cont.)



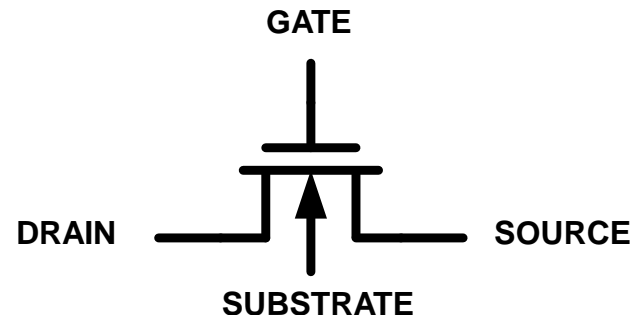
# MOS Transistors

- MOS structure

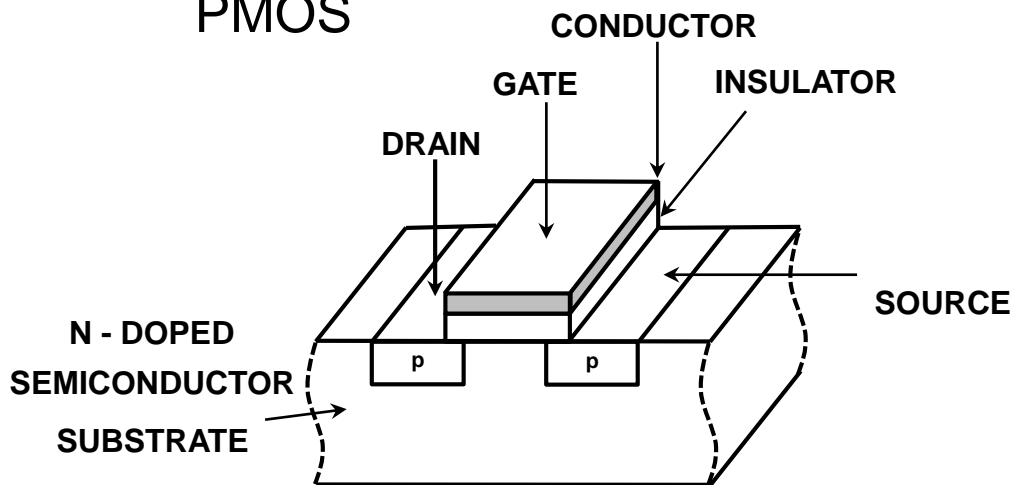
## NMOS



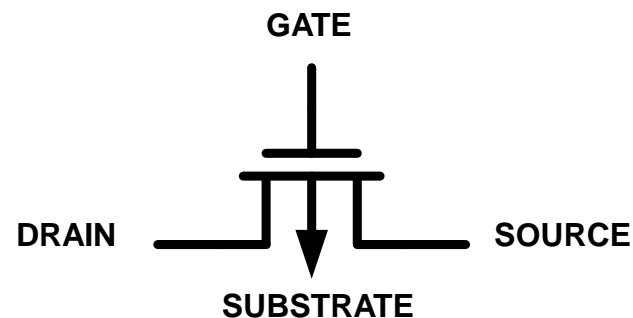
## symbol



## PMOS

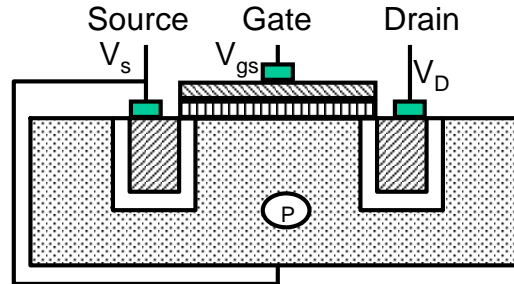


## symbol

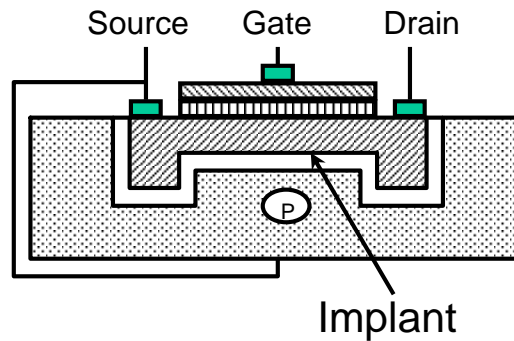


# MOS Transistors (Cont.)

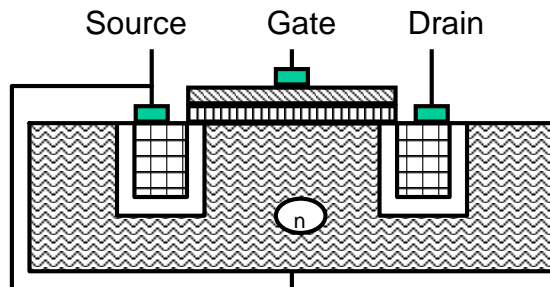
-Enhancement NMOS



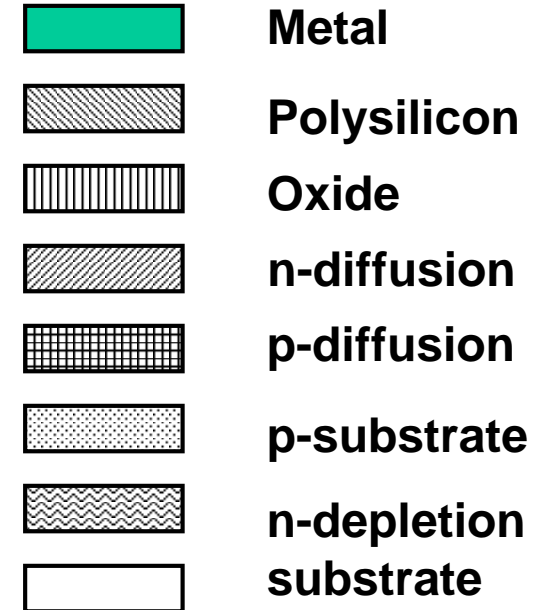
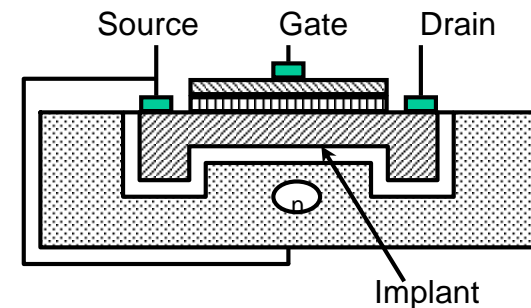
-Depletion NMOS



-Enhancement PMOS

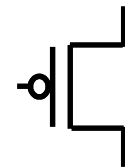
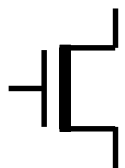
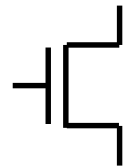
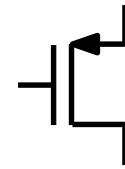
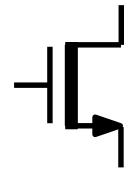
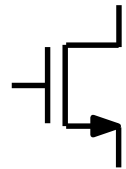
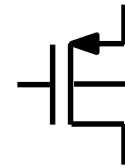
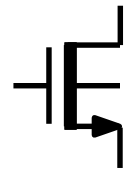
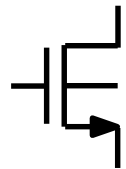
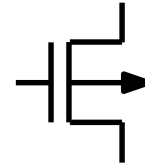
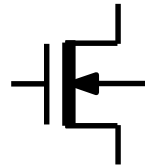
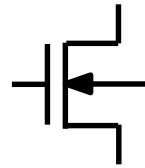


-Depletion PMOS





# MOS Transistor Symbol (Cont.)



nMOS  
enhancement

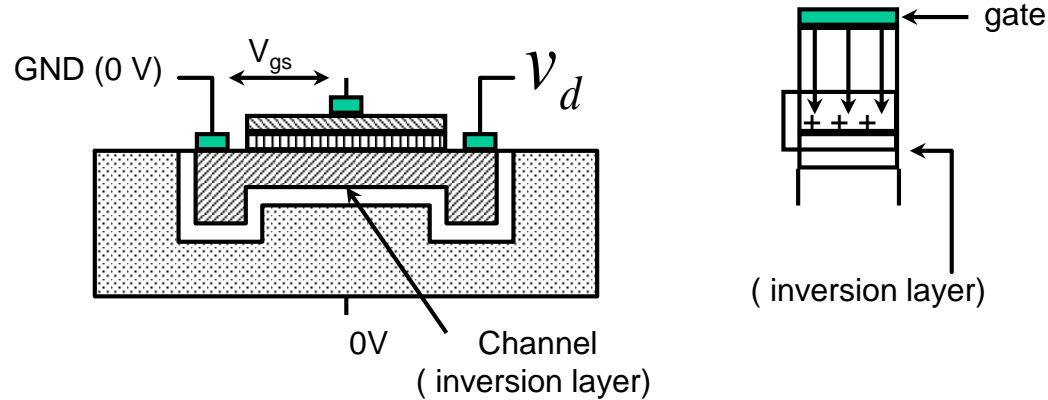
nMOS  
depletion

pMOS  
enhancement

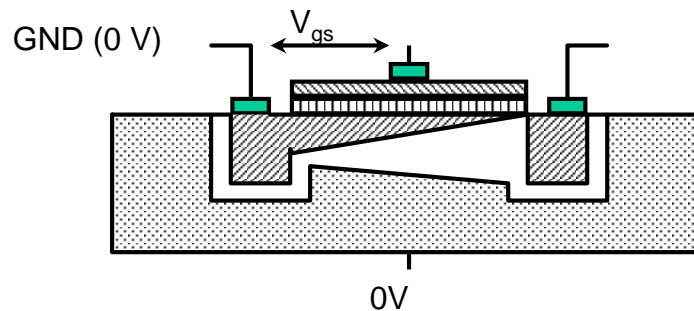
# MOS Transistor Operation

- Example : nMOS

- ◆  $V_{gs} > V_t$ ,  $V_{ds} = 0$  (linear region)



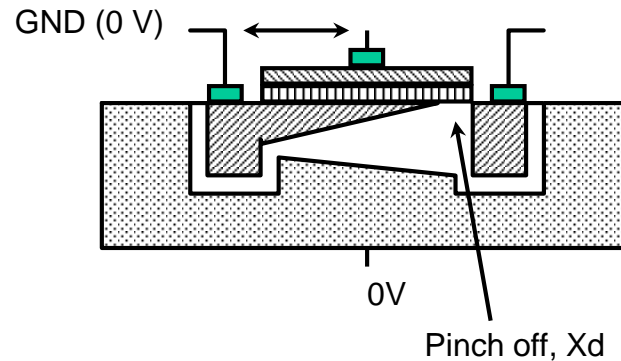
- ◆  $V_{gs} > V_t$ ,  $V_{ds} = V_{gs} - V_t$



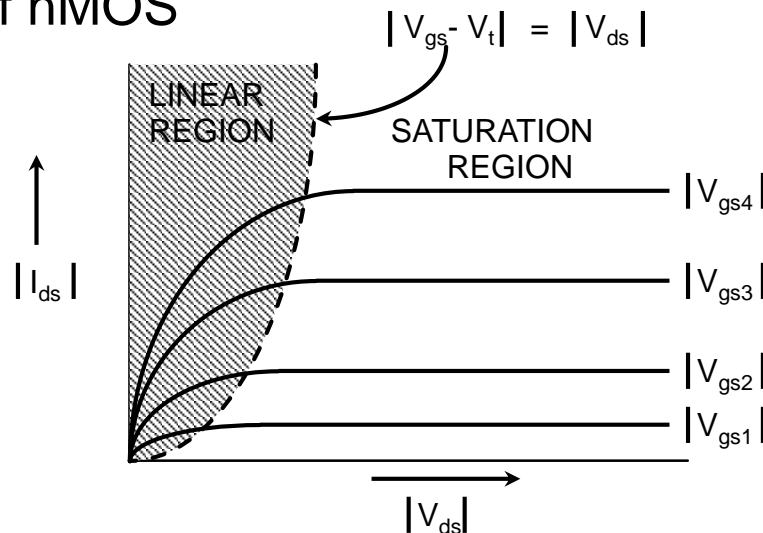
# MOS Transistor Operation (Cont.)

- Example : nMOS

- ◆  $V_{gs} > V_t$ ,  $V_{ds} > V_{gs} - V_t$  (saturation region)



- I-V characteristic of nMOS



# Large Signal Behavior of MOSFETs

- Threshold voltage

$$V_t = V_{t0} + \frac{\sqrt{2q\epsilon_{\text{SiO}_2} N_A}}{C_{\text{ox}}} \left( \sqrt{2\phi_f + V_{\text{SB}}} - \sqrt{2\phi_f} \right)$$

$$= V_{t0} + \gamma \left( \sqrt{2\phi_f + V_{\text{SB}}} - \sqrt{2\phi_f} \right)$$

$$\text{where } \gamma = \frac{\sqrt{2q\epsilon_{\text{SiO}_2} N_A}}{C_{\text{ox}}} \quad \text{and } C_{\text{ox}} = \frac{k_{\text{ox}} \epsilon_0}{t_{\text{ox}}} = \frac{\epsilon_{\text{ox}}}{t_{\text{ox}}}$$

- Large-Signal I-V

$$I_{\text{DS}} = \frac{\mu C_{\text{ox}} W}{2L} (V_{\text{GS}} - V_t)^2 = \frac{kW}{2L} (V_{\text{GS}} - V_t)^2$$

If depletion-layer width  $X_d$  is considered  $L_{\text{eff}} = L - X_d$

$$\Rightarrow I_{\text{DS}} = \frac{kW}{2L_{\text{eff}}} (V_{\text{GS}} - V_t)^2$$

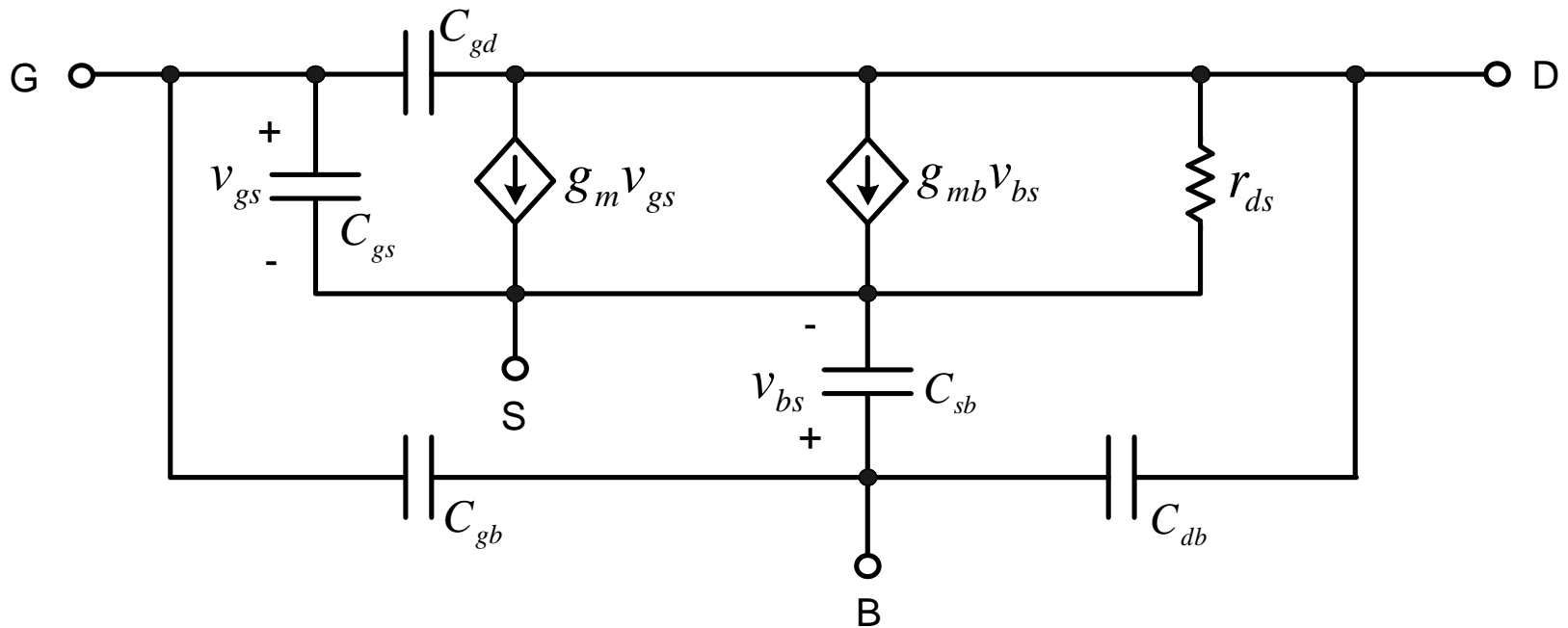
- Channel length modulation

$$\frac{\partial I_{\text{DS}}}{\partial V_{\text{DS}}} = -\frac{kW}{2L_{\text{eff}}^2} (V_{\text{GS}} - V_t)^2 \frac{dL_{\text{eff}}}{dV_{\text{DS}}} = \frac{I_{\text{DS}} dx_d}{L_{\text{eff}} dV_{\text{DS}}}$$

$$\text{Let } \left( \frac{\partial I_{\text{DS}}}{\partial V_{\text{DS}}} \right)^{-1} = r_{ds} = \frac{1}{\lambda I_{\text{DS}}} = \frac{V_A}{I_{\text{DS}}} \Rightarrow \lambda = \frac{1}{V_A} = \frac{1}{L_{\text{eff}}} \left( \frac{dx_d}{dV_{\text{DS}}} \right) \Rightarrow I_{\text{DS}} = \frac{kW}{2L} (V_{\text{GS}} - V_t)^2 (1 + \lambda V_{\text{DS}})$$

# Small Signal Model of MOSFETs in Saturation

- Equivalent circuit model



# Small Signal Model of MOSFETs in Saturation (Cont.)

- $$g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = k' \frac{W}{L} (V_{GS} - V_t) = \sqrt{2k' \frac{W}{L} I_{DS}} \quad \lambda V_{DS} \ll 1$$
- $$g_{mb} = \frac{\partial I_{DS}}{\partial V_{BS}} = k' \frac{W}{L} (V_{GS} - V_t) \frac{\partial V_t}{\partial V_{BS}} = \chi g_m \quad \frac{\partial V_t}{\partial V_{BS}} = \frac{\partial (V_{to} + \gamma \sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f})}{\partial V_{BS}} = \frac{-\gamma}{2\sqrt{2\phi_f + V_{SB}}} = \chi$$
- $$r_{ds} = \left( \frac{\partial I_{DS}}{\partial V_{DS}} \right)^{-1} = \frac{L_{eff}}{I_{DS}} \left( \frac{dX_d}{dV_{DS}} \right)^{-1} = \frac{1}{\lambda I_{DS}} = \frac{V_A}{I_{DS}}$$
- $$C_{sb} = \frac{C_{sb0}}{\left( 1 + \frac{V_{SB}}{\phi_0} \right)^{1/2}}$$
- $$C_{db} = \frac{C_{db0}}{\left( 1 + \frac{V_{DB}}{\phi_0} \right)^{1/2}}$$
- $$C_{gs} = \frac{\partial Q_T}{\partial V_{GS}} \approx \frac{2}{3} W L C_{ox}$$

$$f_t = \frac{g_m}{2\pi C_{gs}} = \frac{1}{2\pi} \frac{K \frac{W}{L} (V_{GS} - V_t)}{\frac{2}{3} W L C_{ox}}$$

$$\omega_t = 2\pi f_t = \frac{3\mu}{2L^2} (V_{GS} - V_t)$$
- Derivation of  $C_{gs}$

◆ Total charge stored in the channel  $Q_T$

$$Q_T = -WC_{ox} \int_0^L [V_{GS} - V(y) - V_t] dy = - \int_0^{V_{GS}-V_t} \frac{W^2 C_{ox}^2 \mu}{I_D} (V_{GS} - V - V_t)^2 dV$$

$$= \frac{2}{3} W L C_{ox} (V_{GS} - V_t)$$

$$C_{gs} = \frac{\partial Q_T}{\partial V_{GS}} = \frac{2}{3} W L C_{ox}$$

## Example—Small Signal Model

- Derive the complete small-signal model for an NMOS transistor with  $I_{DS}=100\mu\text{A}$ ,  $V_{SB}=0.15\text{V}$ ,  $V_{DS}=0.6\text{V}$ . Device parameters are  $2\phi_f = 0.65$ ,  $W=2.5\ \mu\text{m}$ ,  $L=45\ \text{nm}$ ,  $\gamma = 0.45\text{V}^{1/2}$ ,  $\mu_n C_{ox} = 280\mu\text{A}/\text{V}^2$ ,  $\lambda = 2.22\text{V}^{-1}$ ,  $t_{ox} = 1.2\ \text{nm}$ ,  $\Psi_0 = 0.69\ \text{V}$ ,  $C_{sb0} = C_{db0} = 1.125\ \text{fF}$ . Overlap capacitance from gate to source and gate to drain is  $1.25\ \text{fF}$ . Assume  $C_{gb} = 5\text{fF}$ .

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = \sqrt{2 \times 280 \times 10^{-6} \times \frac{2.5}{0.045} \times 100 \times 10^{-6}} \text{ A/V} = 1.76 \text{ mA/V}$$

$$g_{mb} = \gamma \sqrt{\frac{\mu_n C_{ox} \frac{W}{L} I_D}{2(2\phi_f + V_{SB})}} = 0.45 \sqrt{\frac{280 \times 10^{-6} \times \frac{2.5}{0.045} \times 100 \times 10^{-6}}{2 \times (0.65 + 0.15)}} \text{ A/V} = 443 \mu\text{A/V}$$

$$r_{ds} = \frac{1}{\lambda I_{DS}} = \frac{1}{2.22 \times 100 \times 10^{-6}} = 22.2 \text{ k}\Omega$$

## Example—Small Signal Model (Cont.)

- With  $V_{SB}=0.15V$ , we find  $C_{sb} = \frac{C_{sb0}}{\left(1 + \frac{V_{SB}}{\psi_0}\right)^{1/2}} = \frac{1.125}{\left(1 + \frac{0.15}{0.69}\right)^{1/2}} \text{fF} = 1\text{fF}$

- The voltage from drain to body is  $V_{DB} = V_{DS} + V_{SB} = 0.75V$

Hence,  $C_{db} = \frac{C_{db0}}{\left(1 + \frac{V_{DB}}{\psi_0}\right)^{1/2}} = \frac{1.125}{\left(1 + \frac{0.75}{0.69}\right)^{1/2}} \text{fF} = 1.85\text{fF}$

- The oxide capacitance per unit area is

$$C_{ox} = \frac{\epsilon_r \epsilon_{SiO_2}}{t_{ox}} = \frac{3.9 \times 8.854 \times 10^{-14} \text{ F/cm}}{1.2 \times 10^{-9} \text{ m}} \approx 28.7 \text{ fF}/(\mu\text{m})^2$$

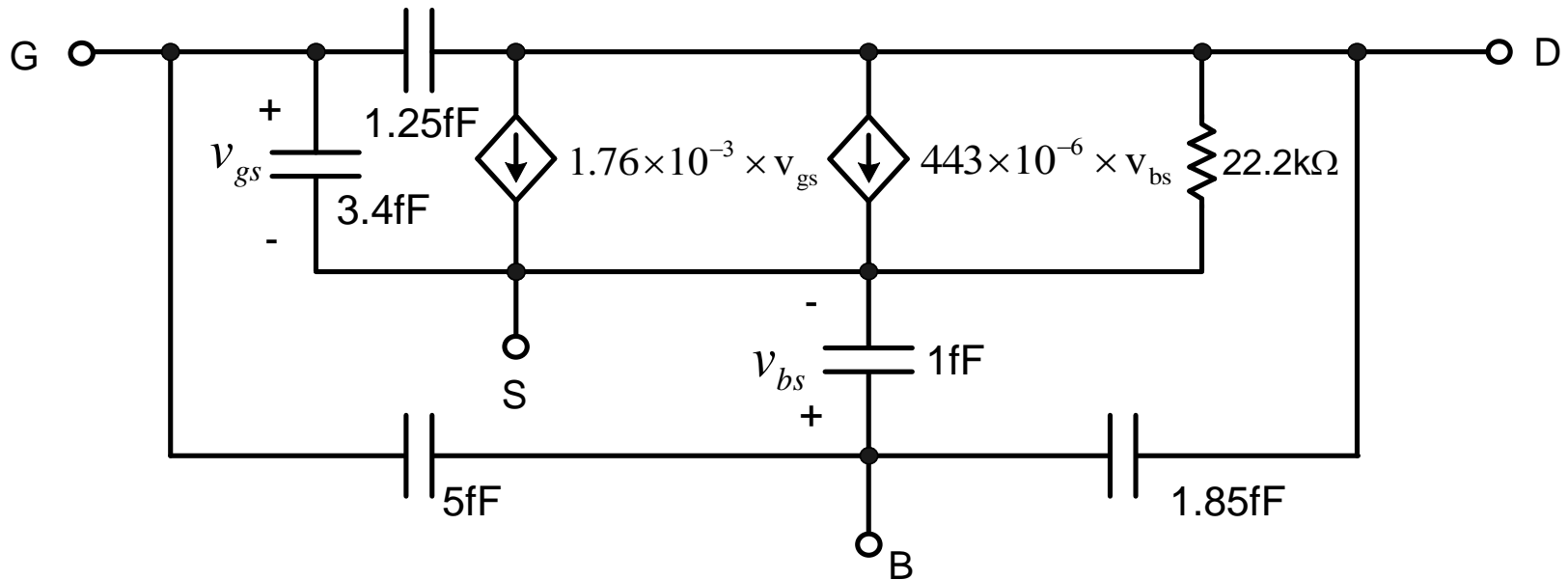
- The intrinsic portion of the gate source capacitance is

$$C_{gs} = \frac{2}{3} \times 2.5 \times 0.045 \times 28.7 \text{ fF} \approx 2.15 \text{ fF}$$



## Example—Small Signal Model (Cont.)

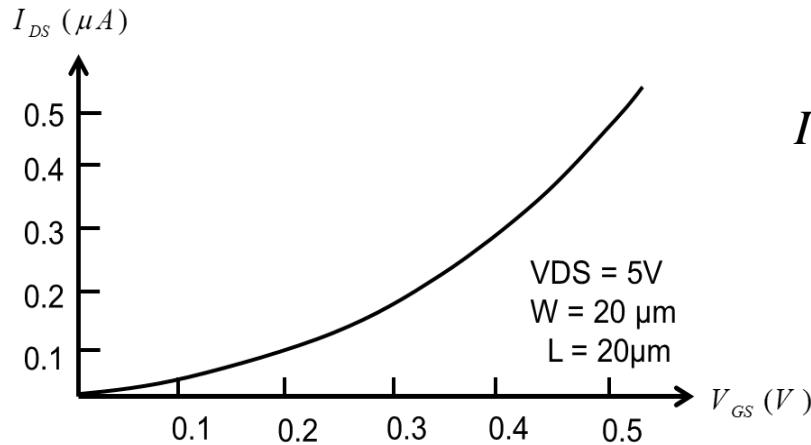
- The addition of overlap capacitance gives  $C_{gs} = 3.4 \text{ fF}$
- Gate-drain capacitance is overlap capacitance  $C_{gd} = 1.25 \text{ fF}$
- The complete small-signal equivalent circuit is shown below



- The  $f_T$  of the device can be calculated with  $C_{gb} = 5 \text{ fF}$  giving

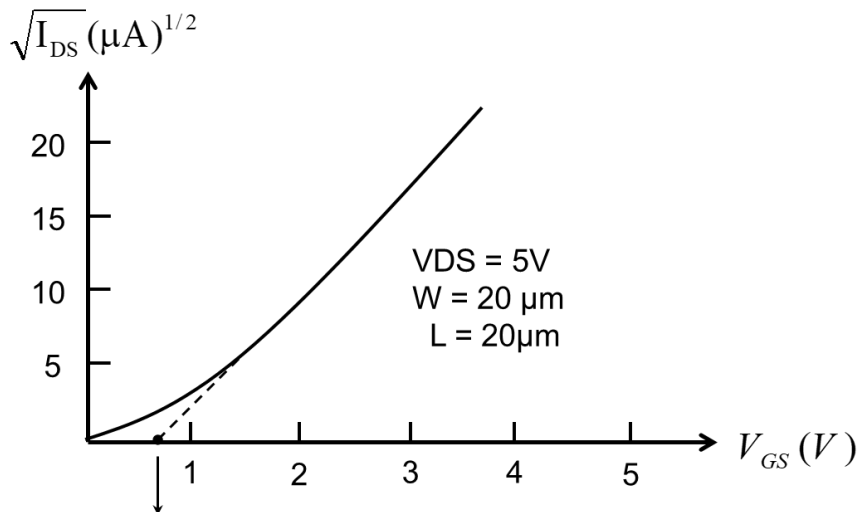
$$f_T = \frac{1}{2\pi} \frac{g_m}{C_{gs} + C_{gd} + C_{gb}} = \frac{1}{2\pi} \frac{1.76 \times 10^{-3}}{(3.4 + 1.25 + 5) \times 10^{-15}} \cong 29 \text{ GHz}$$

# Subthreshold Conduction in MOSFETs

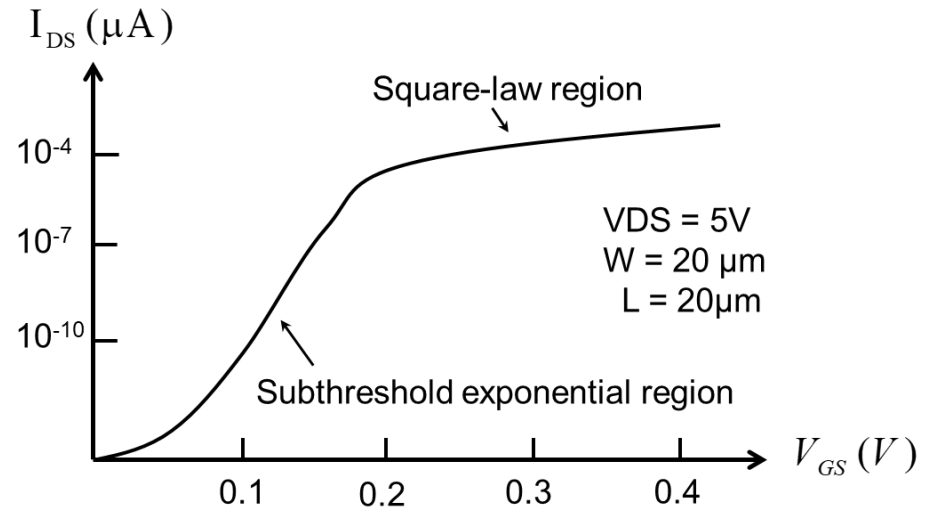


$$I_{DS} = k_x \frac{W}{L} \exp^{\frac{V_{eff}}{nV_t}} \left( 1 - \exp^{\frac{-V_{DS}}{nV_t}} \right)$$

where  $k_x$  depends on process parameters  
 $n \approx 1.5$



Plot on linear scales as  $\sqrt{I_{DS}}$  versus  $V_{GS}$   
 → Show **square-law** characteristic



Plot on log-linear scales  
 → Show **exponential** characteristic  
 in the subthreshold region

# Mobility Degradation

- Large lateral electric fields accelerate carriers up to a maximum velocity
- Larger vertical electric fields  $\rightarrow$  effective channel depth  $\downarrow \rightarrow$  collisions  $\uparrow$
- These effects can be modeled by an effective carrier mobility

$$\mu_{n,\text{eff}} \approx \frac{\mu_n}{[1 + (\theta V_{\text{eff}})^m]^{1/m}}$$

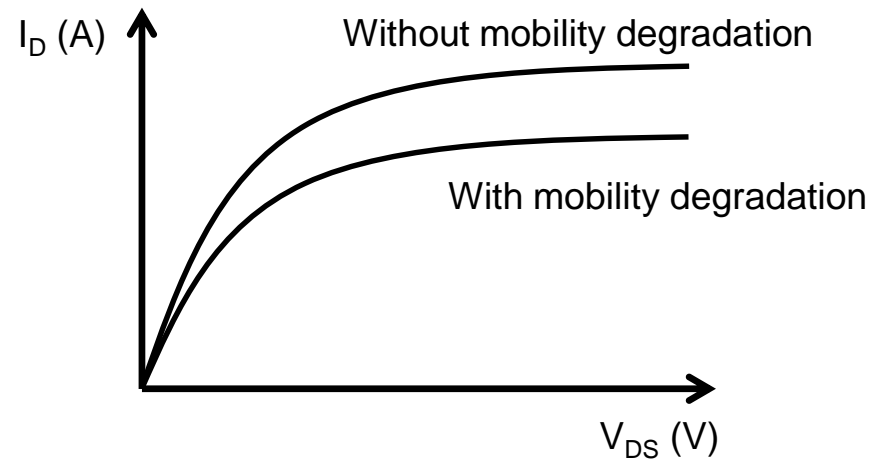
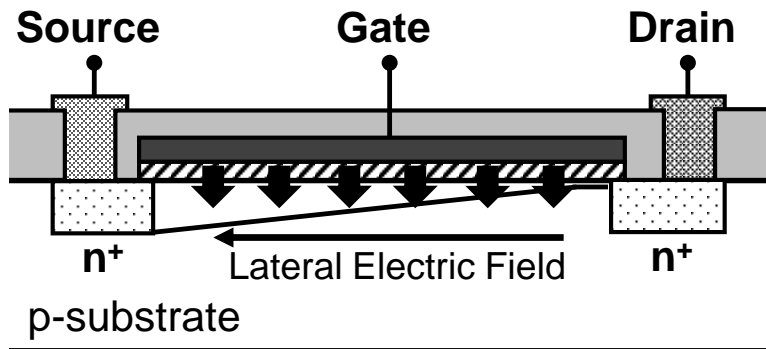
$$I_D = \frac{1}{2} \mu_n C_{\text{ox}} \frac{W}{L} V_{\text{eff}}^2 \left( \frac{1}{[1 + (\theta V_{\text{eff}})^m]^{1/m}} \right)$$

where  $\theta$  and  $m$  are device parameters



$$I_D = \frac{1}{2} \mu_n C_{\text{ox}} \frac{W}{L} V_{\text{eff}}^\alpha, \quad \alpha < 2$$

This effect can also be expressed as  $\alpha$ -law model from curve-fitting



# Substrate Current Flow in MOSFETs

$$I_{DB} = k_1(V_{DS} - V_{DS(sat)})I_{DS}\exp\left[-\frac{k_2}{(V_{DS} - V_{DS(sat)})}\right]$$

where  $k_1$  and  $k_2$  are process-dependent parameters and  $V_{DSsat}$  is the value of  $V_{DS}$  where the drain characteristics enter the saturation region

$$g_{db} = \frac{\partial I_{DB}}{\partial V_{DB}} = \frac{k_2 I_{DB}}{(V_{DS} - V_{DS(sat)})^2}$$

## Example (1/2)

- Calculate  $r_{db} = 1/g_{db}$  for  $V_{DS} = 2\text{ V}$  and  $4\text{ V}$ , and compare with the device  $r_{ds}$ .

Assume  $I_{DS} = 100\mu\text{A}$ ,  $\lambda = 0.45\text{ V}^{-1}$ ,  $V_{DS(\text{sat})} = 0.3\text{ V}$ ,  $K1 = 5\text{ V}^{-1}$ , and  $K2 = 30\text{ V}$ .

For  $V_{DS} = 2\text{ V}$ , we have

$$I_{DB} = 5 \times 1.7 \times 100 \times 10^{-6} \times \exp\left(-\frac{30}{1.7}\right) = 1.8 \times 10^{-11}\text{ A}$$

$$g_{db} = \frac{30 \times 1.8 \times 10^{-11}}{1.7^2} = 1.9 \times 10^{-10} \frac{\text{A}}{\text{V}}$$

and thus

$$r_{db} = \frac{1}{g_{db}} = 5.3 \times 10^9\ \Omega = 5.3\ \text{G}\Omega$$

## Example (2/2)

This result is negligibly large compared with

$$r_{ds} = \frac{1}{\lambda I_D} = \frac{1}{0.45 \times 100 \times 10^{-6}} = 22.2 \text{ k}\Omega$$

However, for  $V_{DS} = 4 \text{ V}$

$$I_{DB} = 5 \times 3.7 \times 100 \times 10^{-6} \times \exp\left(-\frac{30}{3.7}\right) = 5.6 \times 10^{-7} \text{ A}$$

The substrate leakage current is now about 0.5% of the drain current.

We find

$$g_{db} = \frac{30 \times 5.6 \times 10^{-7}}{3.7^2} = 1.2 \times 10^{-6} \frac{\text{A}}{\text{V}}$$

and thus

$$r_{db} = \frac{1}{g_{db}} = 8.33 \times 10^5 \text{ }\Omega = 833 \text{ k}\Omega$$

This parasitic resistor is now comparable to  $r_{ds}$  and can have a dominant effect on high-output-impedance MOS current mirrors.

# Summary of MOSFET Parameters

- Large-Signal Operation

Quantity	Formula
Drain current (saturation region)	$I_{ds} = \frac{\mu C_{ox}}{2} \frac{W}{L} (V_{gs} - V_t)^2 (1 + \lambda V_{ds})$
Drain current (triode region)	$I_{ds} = \frac{\mu C_{ox}}{2} \frac{W}{L} [2(V_{gs} - V_t)V_{ds} - V_{ds}^2]$
Threshold voltage	$V_t = V_{t0} + \gamma[\sqrt{2\phi_f - V_{sb}} - \sqrt{2\phi_f}]$
Threshold voltage parameter	$\gamma = \frac{1}{C_{ox}} \sqrt{2q\epsilon N_A}$
Oxide capacitance	$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = 3.45 \text{ fF}/\mu\text{m}^2 \text{ for } t_{ox} = 100 \text{ \AA}$

# Summary of MOSFET Parameters

- Large-Signal Operation

Quantity	Formula
Top-gate transconductance	$g_m = \mu C_{ox} \frac{W}{L} (V_{GS} - V_t) = \sqrt{2I_{DS} \mu C_{ox} \frac{W}{L}}$
Transconductance-to-current ratio	$\frac{g_m}{I_{DS}} = \frac{2}{V_{GS} - V_t}$
Body-effect transconductance	$g_{mb} = \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}} g_m = \chi g_m$
Channel-length modulation parameter	$\lambda = \frac{1}{V_A} = \frac{1}{V_{eff}} \frac{dX_d}{dV_{DS}}$
Output resistance	$r_{ds} = \frac{1}{\lambda I_{DS}} = \frac{L_{eff}}{I_{DS}} \left( \frac{dX_d}{dV_{DS}} \right)^{-1}$



# Summary of MOSFET Parameters

Quantity	Formula
Effective channel length	$L_{\text{eff}} = L_{\text{drwn}} - 2L_d - X_d$
Maximum gain	$g_m r_{ds} = \frac{1}{\lambda} \frac{2}{V_{GS} - V_t} = \frac{2V_A}{V_{GS} - V_t}$
Source-body depletion capacitance	$C_{sb} = \frac{C_{sb0}}{\left(1 + \frac{V_{SB}}{\psi_0}\right)^{0.5}}$
Drain-body depletion capacitance	$C_{db} = \frac{C_{db0}}{\left(1 + \frac{V_{DB}}{\psi_0}\right)^{0.5}}$
Gate-source capacitance	$C_{gs} = \frac{2}{3} WLC_{ox}$
Transition frequency	$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd} + C_{gb})}$

# SPICE MOSFET Model Parameters of A Typical NMOS Process (MOSIS)

Parameter (Level 2 model)	Enhancement	Depletion	Units
VTO	1.14	-3.79	V
KP	37.3	32.8	$\mu\text{A}/\text{V}^2$
GAMMA	0.629	0.372	$\text{V}^{1/2}$
PHI	0.6	0.6	V
LAMBDA	3.1E-2	1.00E-6	$\text{V}^{-1}$
CGSO	1.60E-4	1.60E-4	fF/ $\mu$ width
CGDO	1.60E-4	1.60E-4	fF/ $\mu$ width
CGBO	1.70E-4	1.70E-4	fF/ $\mu$ width
RSH	25.4	25.4	$\Omega/\square$
CJ	1.1E-4	1.1E-4	pF/ $\mu^2$
MJ	0.5	0.5	
CJSW	5.0E-4	5.0E-4	pF/ $\mu^2$ perimeter
MJSW	0.33	0.33	
TOX	544	544	$\text{\AA}$

# SPICE MOSFET Model Parameters of A Typical NMOS Process (MOSIS) (Cont.)

(Level 2 model)	Enhancement	Depletion	Units
<b>NSUB</b>	<b>2.09E15</b>	<b>1.0E16</b>	<b>1/cm<sup>2</sup></b>
<b>NSS</b>	<b>0</b>	<b>0</b>	<b>1/cm<sup>3</sup></b>
<b>NFS</b>	<b>1.90E12</b>	<b>4.3E12</b>	<b>1/cm<sup>2</sup></b>
<b>TPG</b>	<b>1</b>	<b>1</b>	
<b>XJ</b>	<b>1.31</b>	<b>0.6</b>	
<b>LD</b>	<b>0.826</b>	<b>1.016</b>	<b><math>\mu</math></b>
<b>UO</b>	<b>300</b>	<b>900</b>	<b><math>\mu</math></b>
<b>UCRIT</b>	<b>1.0E6</b>	<b>0.805E6</b>	<b>cm<sup>2</sup>/(v · s)</b>
<b>UEXP</b>	<b>1.001E-3</b>	<b>1.001E-3</b>	<b>V/cm</b>
<b>VMAX</b>	<b>1.0E5</b>	<b>6.75E5</b>	<b>m/s</b>
<b>NEFF</b>	<b>1.001E-2</b>	<b>1.001E-2</b>	
<b>DELTA</b>	<b>1.16</b>	<b>2.80</b>	

- The SPICE parameters: Empirical parameters
  - ◆ Fitting measured device characteristics to the mathematical equations
  - ◆ Using a numerical optimization algorithm
- This approach gives good fit to the model but causes a deviation from the typical parameters.
- Parameter relationships may not be self-consistent with some of the fundamental relationships.

\*Please refer to the chapters about SPICE model in the HSPICE document suggested in assignment 1

# Appendix

- Resistance Estimation
- Capacitance Estimation
- Inductance Estimation

# Resistance Estimation

- Sheet Resistance

- ◆  $R = \rho L / A = \rho L / tW = R_{\square} L / W$

- ◆  $R_{\square} = \rho / t$

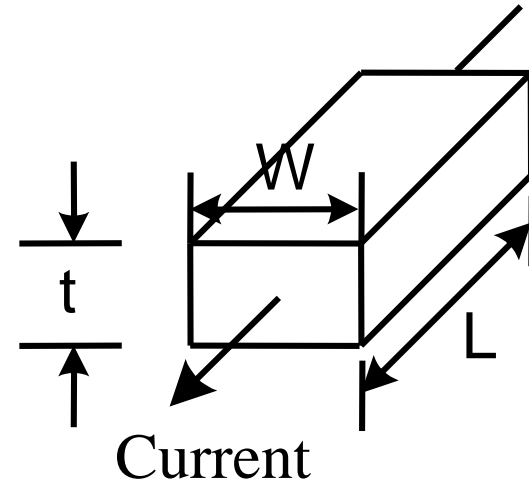
$\rho$ : resistivity

$t$ : thickness

$L$ : conductor length

$W$ : conductor width

$R_{\square}$ : sheet resistance (ohm/square,  $\Omega / \square$ )



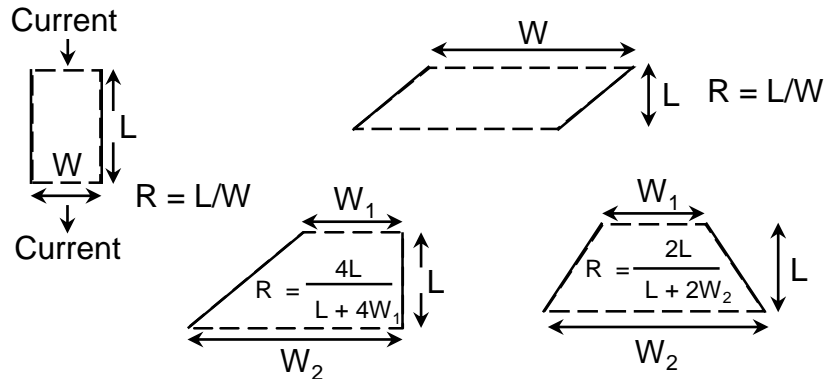
# Resistance Estimation (cont.)

- Typical sheet resistance for conductors

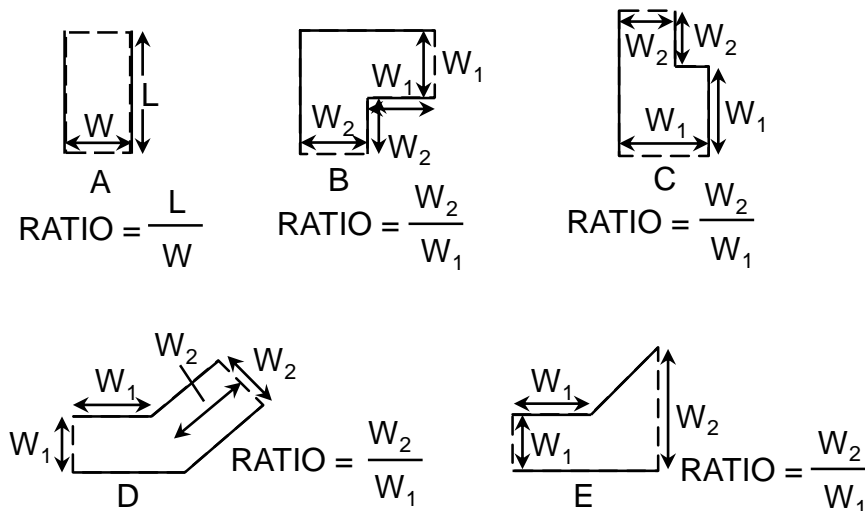
Material	Min.	Typical	Max.
Intermetal (metal1-metal2)	0.05	0.07	0.1
Top-metal(metal3)	0.03	0.04	0.05
Polysilicon	15	20	30
Silicide	2	3	6
Diffusion(n <sup>+</sup> , p <sup>+</sup> )	10	25	100
Silicided diffusion	2	4	10
N-well	1K	2K	5K

# Resistance Estimation of Nonrectangular Shapes

## ● Direct estimation



## ● Table-assisted estimation



Shape	Ratio	Resistance
A	1	1
A	5	5
B	5	5
B	1	2.5
B	2	2.55
B	3	2.66
C	1.5	2.1
C	2	2.25
C	3	2.5
C	4	2.65
D	1	2.2
D	1.5	2.3
D	2	2.3
D	3	2.6
E	1.5	1.45
E	2	1.8
E	3	2.3
E	4	2.65

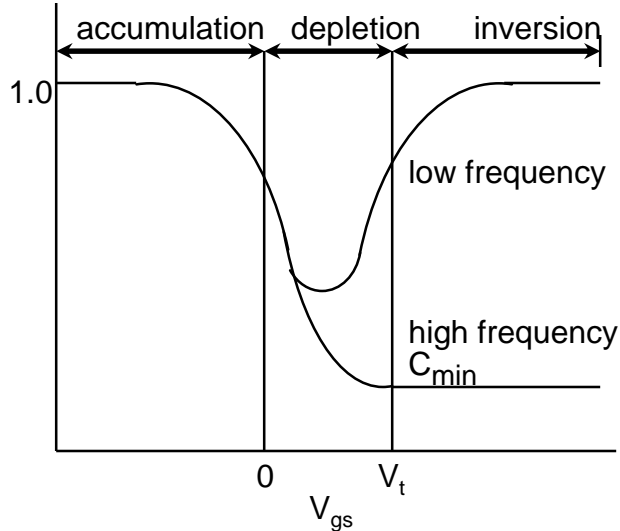
# Contact and Via Resistance

- Proportional to the area of the contact
  - ◆ e.g. feature size↓ =>Rcontact↑
- $0.25\Omega$  ~ a few tens of  $\Omega$ s
- Multiple contacts to obtain low-resistance interlayer connections



# MOS-Capacitor Characteristics

## ● C-V plot



Three regions in the plot

- (i) Accumulation region
- (ii) Depletion region
- (iii) Inversion region

## ● Accumulation region

$$C_o = \frac{\epsilon_{\text{SiO}_2} \epsilon_o}{t_{\text{ox}}} A = C_{\text{ox}} \bullet A$$

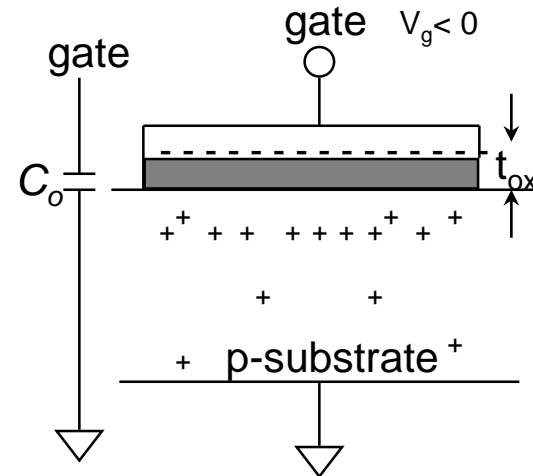
$C_o$  : gate capacitance

$\epsilon_{\text{SiO}_2}$  : dielectric constant of  $\text{SiO}_2$  (= 3.9)

$\epsilon_o$  : permittivity of free space

$A$  : gate area

$$C_{\text{ox}} = \frac{\epsilon_{\text{SiO}_2} \epsilon_o}{t_{\text{ox}}} : \text{gate capacitance per unit area}$$



# MOS-Capacitor Characteristics (cont.)

## ● Depletion region

$$C_{\text{dep}} = \frac{\epsilon_{\text{Si}} \epsilon_0}{d} A = C_{\text{ox}}$$

$d$  : depletion layer depth

$\epsilon_{\text{Si}}$  : dielectric constant of Si (= 12)

$$C_{\text{gb}} = \frac{C_o C_{\text{dep}}}{C_o + C_{\text{dep}}} : \text{gate capacitance per unit area}$$

Where  $C_o$  is low frequency capacitance between gate and surface

## ● Inversion region

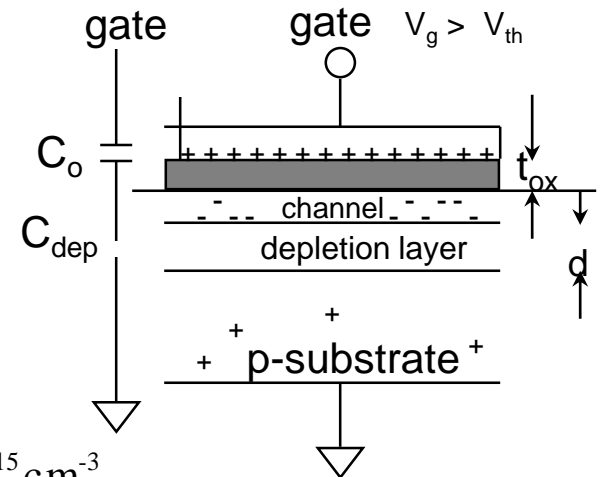
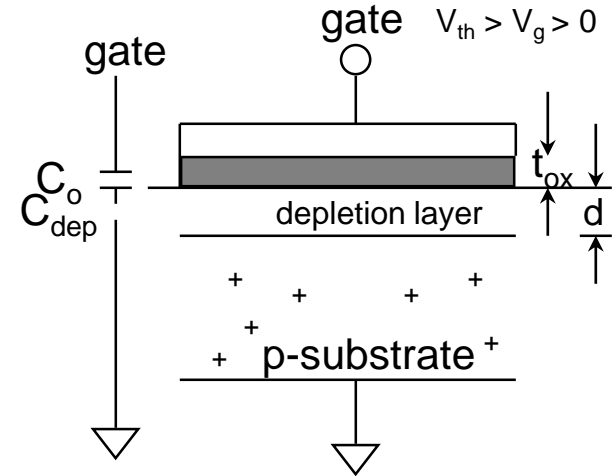
$$C_{\text{gb}} = \begin{cases} C_o : \text{static (i.e. low frequency, } < 100\text{Hz)} \\ \frac{C_o C_{\text{dep}}}{C_o + C_{\text{dep}}} = C_{\text{min}} : \text{dynamic (i.e. high frequency)} \end{cases}$$

$C_{\text{min}}$

–  $C_{\text{dep}}$  depends on the depth of the depletion region, i.e. depends on substrate doping density.

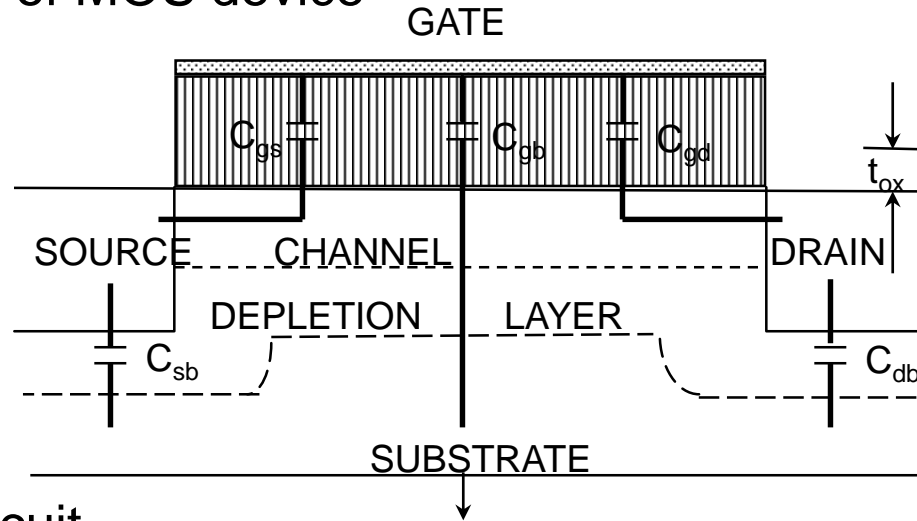
– For  $t_{\text{ox}} = 100 \sim 200 \text{ \AA}$ ,  $C_{\text{min}}/C_o$  varies from 0.02 ~ 0.3

for substrate doping density varies from  $1 \times 10^{14} \text{ cm}^{-3}$  to  $5 \times 10^{15} \text{ cm}^{-3}$

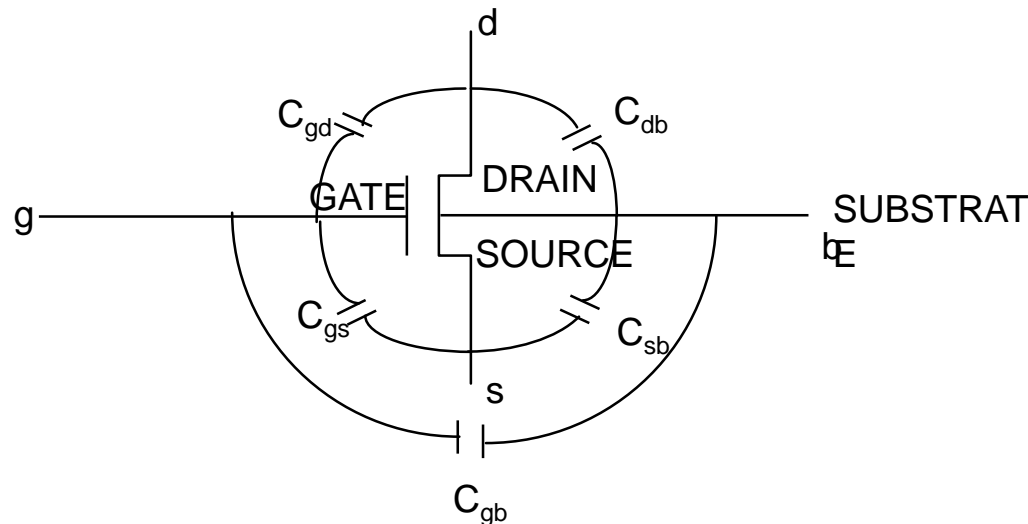


# MOS Device Capacitance

- Cross section of MOS device



- Equivalent circuit



# MOS Device Capacitance (cont.)

- Approximation of gate capacitance

— Self-aligned process is assumed (i.e. overlap caps. are negligible)

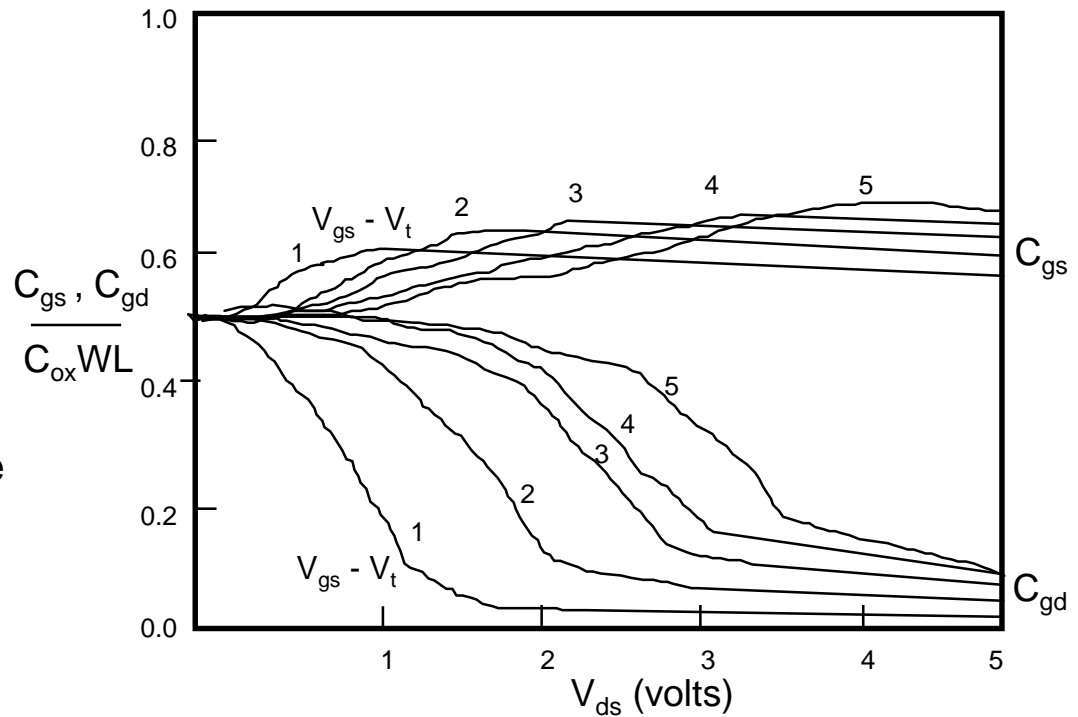
Parameter	off	Non-saturated	Saturated
$C_{gb}$	$\frac{\epsilon A}{t_{ox}}$	0	0
$C_{gs}$	0	$\frac{\epsilon A}{2t_{ox}}$	$\frac{2\epsilon A}{3t_{ox}}$
$C_{gd}$	0	$\frac{\epsilon A}{2t_{ox}}$	0 (finite for short channel devices)
$C_g = C_{gb} + C_{gs} + C_{gd}$	$\frac{\epsilon A}{t_{ox}}$	$\frac{\epsilon A}{t_{ox}}$	$\frac{2\epsilon A}{3t_{ox}} \rightarrow \frac{0.9 \epsilon A}{t_{ox}}$ (short channel)

# C<sub>gs</sub>, C<sub>gd</sub> and C<sub>ox</sub>

- Example 1: W=49.2μm, L=4.5μm (long channel)
  - ◆ C<sub>gs</sub> and C<sub>gd</sub>

large L  
 \*large C<sub>g</sub> & small C<sub>gd</sub>  
 (in saturation region)  

$$\frac{C_{gd}}{C_g} \approx 0$$
  
 (C<sub>gd</sub> is due to channel side fringing fields between gate and drain.)

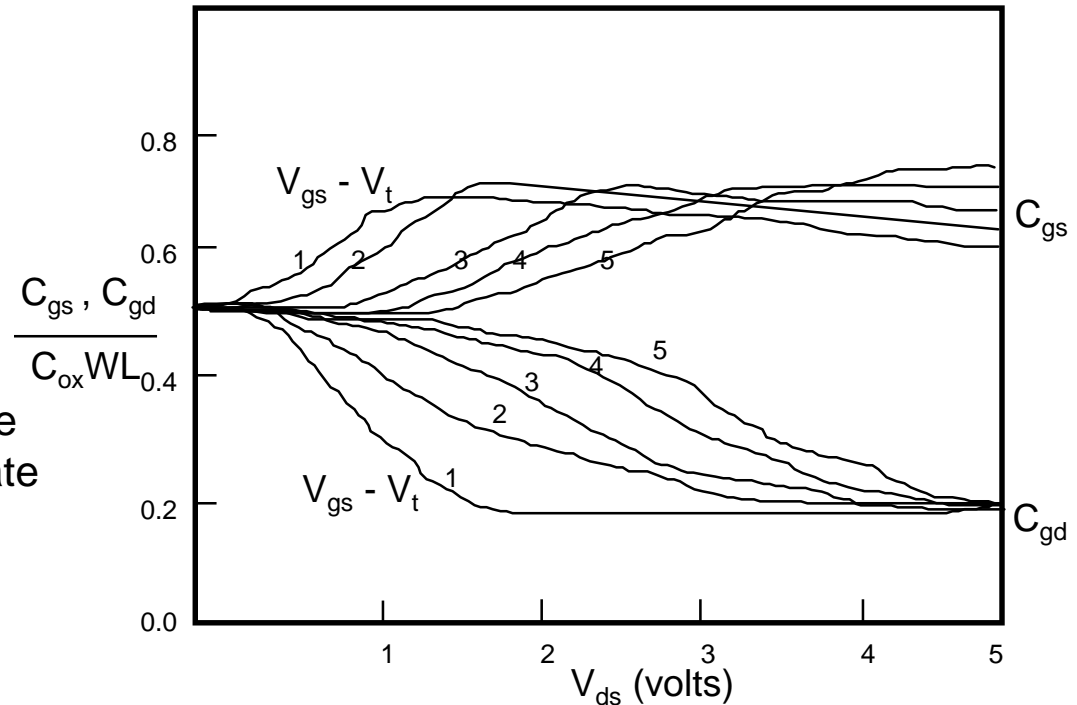


# C<sub>gs</sub>, C<sub>gd</sub> and C<sub>ox</sub> (cont.)

- Example 2: L=0.75μm (short channel)

- ◆ C<sub>gs</sub> and C<sub>gd</sub>

small L  
 \*small C<sub>g</sub> & small C<sub>gd</sub>  
 (in saturation region)  
 $\frac{C_{gd}}{C_g} \approx 0.2$   
 (C<sub>gd</sub> is due to channel side fringing fields between gate and drain.)

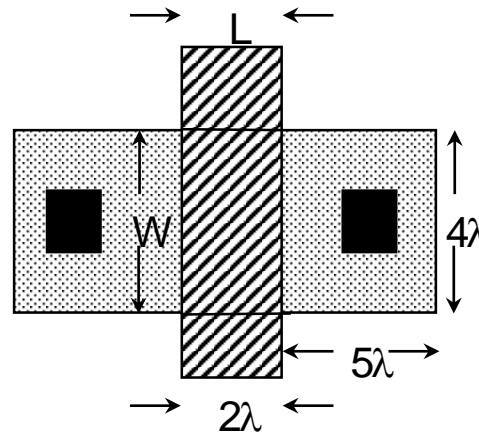


# Cox, gate capacitance per unit area

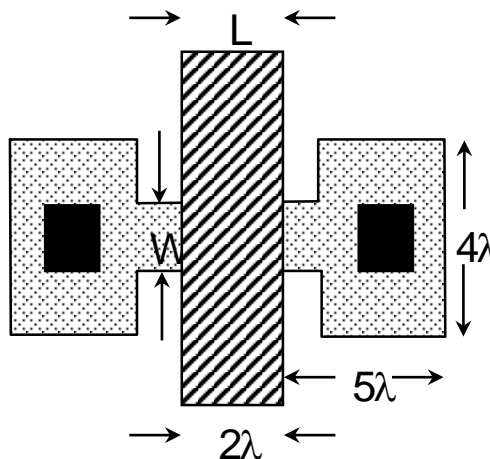
- $C_{ox} = \frac{\epsilon_{SiO_2} \epsilon_0}{t_{ox}} A$ ; where  $\epsilon_{SiO_2} = 3.9$  and  $\epsilon_0 = 8.854 \times 10^{-14}$   
e.g.  $t_{ox} = 350 \text{ \AA} \Rightarrow C_{ox} \approx 1 \times 10^{-3} \text{ pF}/\mu\text{m}^2 = 1 \text{ fF}/\mu\text{m}^2$

- Unit transistor

- ◆ It is the same width as a metal-diffusion contact



- Minimum-size transistor



# Diffusion Capacitance

- Area and periphery

$$C_d = C_{ja} \cdot (ab) + C_{jp} \cdot (2a + 2b)$$

$C_{ja}$ : junction capacitance per  $\mu\text{m}^2$

$C_{jp}$ : periphery capacitance per  $\mu\text{m}$

$a$ : width of diffusion region

$b$ : length of diffusion region

- Typical value (1  $\mu\text{m}$  n-well process)

$C_{ja}$ :  $2 \cdot 10^{-4} \text{ pf}/\mu\text{m}^2$  (n+ diffusion)

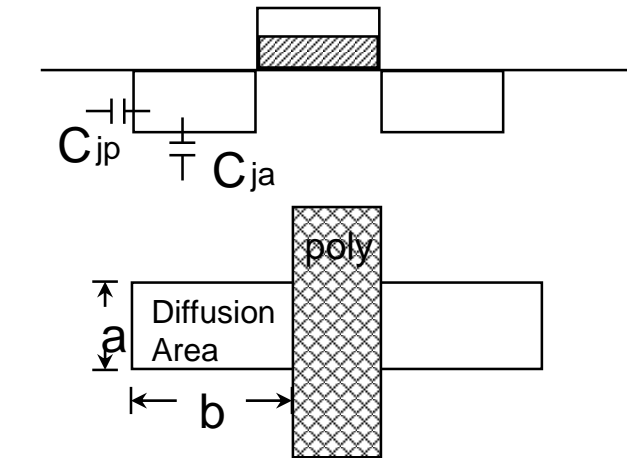
$5 \cdot 10^{-4} \text{ pf}/\mu\text{m}^2$  (p+ diffusion)

$C_{jp}$ :  $4 \cdot 10^{-4} \text{ pf}/\mu\text{m}^2$  (n+ diffusion)

$4 \cdot 10^{-4} \text{ pf}/\mu\text{m}^2$  (p+ diffusion)

- Voltage dependent

$$C_j(V_j) = C_{j0} \left( 1 - \frac{V_j}{V_b} \right)^{-m}$$



$V_j$  is junction voltage

$V_b$  is built-in junction potential  $\sim 0.6\text{V}$

$C_{j0}$  is zero bias capacitance

$m = 0.3$  (graded junction)  $\sim$

$0.5$  (abrupt junction)



# SPICE Modeling of MOS Capacitances

## ● SPICE example

```
M14350NFET W = 4UL = 1U AS = 15P AD = 15PPS = 11.5UPD = 11.5U
```

```
.MODEL NFET NMOS
```

```
+ TOX = 100E-8
```

```
+ CGBO = 200P CGSO = 600P CGDO = 600P
```

```
+ CJ = 200UCJSW = 400PMJ = 0.5MJSW = 0.3PB = 0.7
```

```
+ .....
```

node4- drain	$TOX = 100\text{\AA}$	$C_{gbo}$ occurs due to the polysilicon extension beyond the channel ( $200 \times 10^{-12} \text{F/M}$ )
node3- gate	source area $AS = 15\mu\text{m}^2$	$C_{gso}$ and $C_{gdo}$ represent the gate-to-source/drain capacitance due to overlap in the physical structure of the transistor ( $600 \times 10^{-12} \text{F/M}$ )
node5- source	drain area $AD = 15\mu\text{m}^2$	
node0- substrate	source periphery $PS = 11.5\mu\text{m}$	
channel width = $4\mu\text{m}$	drain periphery $PD = 11.5\mu\text{m}$	
channel length = $1\mu\text{m}$		

# SPICE Modeling of MOS Capacitances (cont.)

## ● Capacitance

gate capacitance

$$C_{g(\text{intrinsic})} = W \cdot L \cdot C_{\text{OX}} = 4 \times 1 \times 35 \times 10^{-4} \text{PF} = 0.014 \text{PF}$$

$$\begin{aligned} C_{g(\text{extrinsic})} &= (W \cdot C_{\text{gs0}}) + (W \cdot C_{\text{gd0}}) + (2L \cdot C_{\text{gbo}}) \\ &= 4 \times 6 \times 10^{-4} + 4 \times 6 \times 10^{-4} + 2 \times (1 \times 2 \times 10^{-4}) \text{PF} \\ &= 0.0052 \text{PF} \end{aligned}$$

$$C_{g(\text{total})} = C_{g(\text{intrinsic})} + C_{g(\text{extrinsic})} \approx 0.02 \text{PF}$$

source and drain capacitance

$$C_j = \left( \text{Area} \cdot C_j \cdot \left( 1 + \frac{VJ}{PB} \right)^{-MJ} \right) + \left( \text{periphery} \cdot C_{\text{JSW}} \cdot \left( 1 + \frac{VJ}{PB} \right)^{-M_{\text{JSW}}} \right)$$

where

$C_j$  = the zero-bias capacitance per junction area

$C_{\text{JSW}}$  = the zero-bias capacitance per junction periphery

$MJ$  = the grading coefficient of the junction bottom

$M_{\text{JSW}}$  = the grading coefficient of the junction sidewall

$VJ$  = the junction potential

$PB$  = the built-in voltage (~ 0.4 - 0.8 volts)

Area =  $A_S$  or  $A_D$ , the area of the source or drain

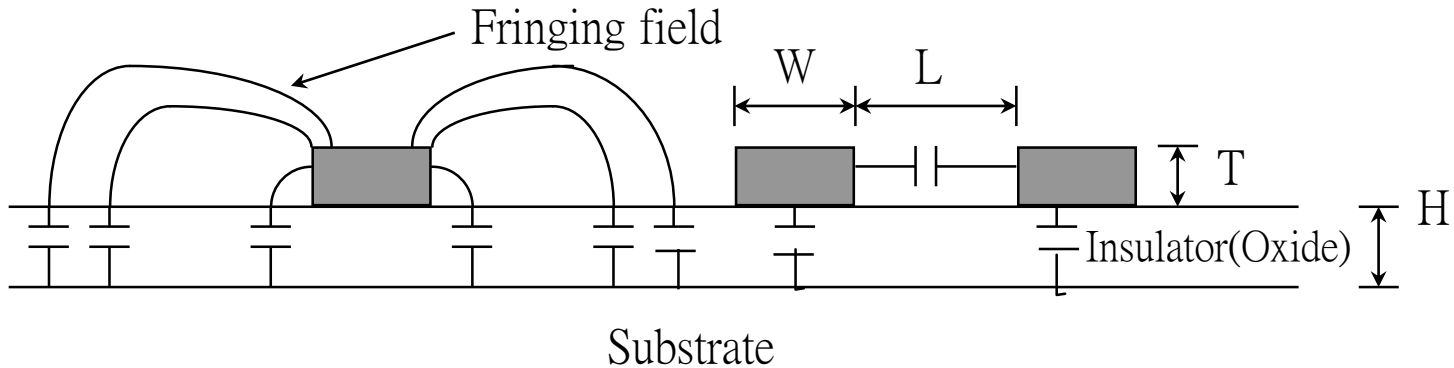
Periphery =  $P_S$  or  $P_D$ , the periphery of the source or drain

$$C_{j(\text{drain})} = 0.0043 \text{PF} \quad (VJ = 2.5 \text{V is assumed})$$

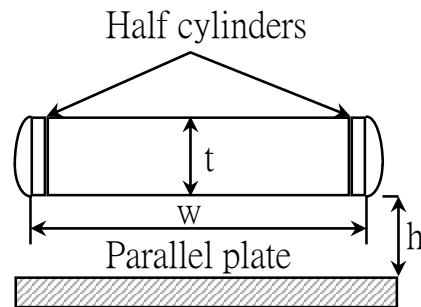
$$C_{j(\text{source})} = 0.0043 \text{PF} \quad (VJ = 2.5 \text{V is assumed})$$

# Routing Capacitance

- Single wire capacitance
  - ◆ Parallel-plate effect and fringing effect



- ◆ Accurate capacitance evaluation : use computer
- ◆ Hand calculation : use simple model (less than 10% error)

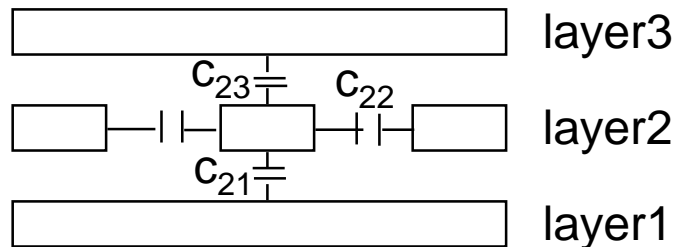


$$C = \epsilon \left[ \left( \frac{w}{h} \right) + 0.77 + 1.06 \left( \frac{w}{h} \right)^{0.25} + 1.06 \left( \frac{t}{h} \right)^{0.5} \right]$$

# Routing Capacitance (cont.)

- Multiple conductor capacitances

- ◆ Three-layer example



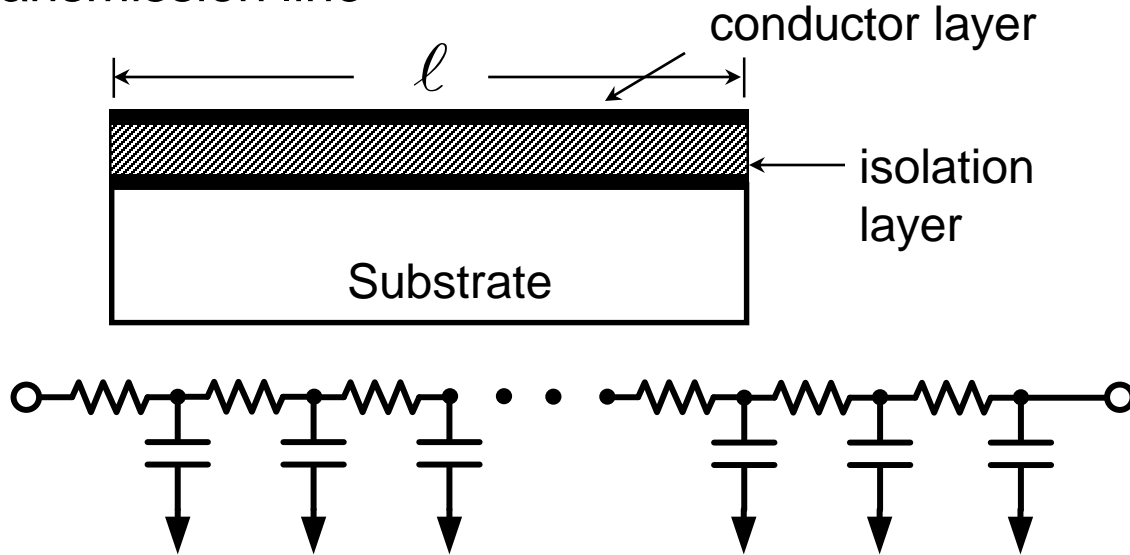
Capacitance calculation is very complex—refer to textbook

- ◆ Typical dielectric and conductor thicknesses

Thin-oxide	200Å	Metal1	6000Å
Field-oxide	6000Å	M <sub>1</sub> -M <sub>2</sub> oxide	6000Å
Polysilicon	3000Å	Metal2	12000Å
M <sub>1</sub> -poly-oxide	6000Å	Passivation	20000Å

# Distributed RC Effects

- Transmission line



- Delay time from one end to the other end

$$t \cong \frac{rc}{2} l^2$$

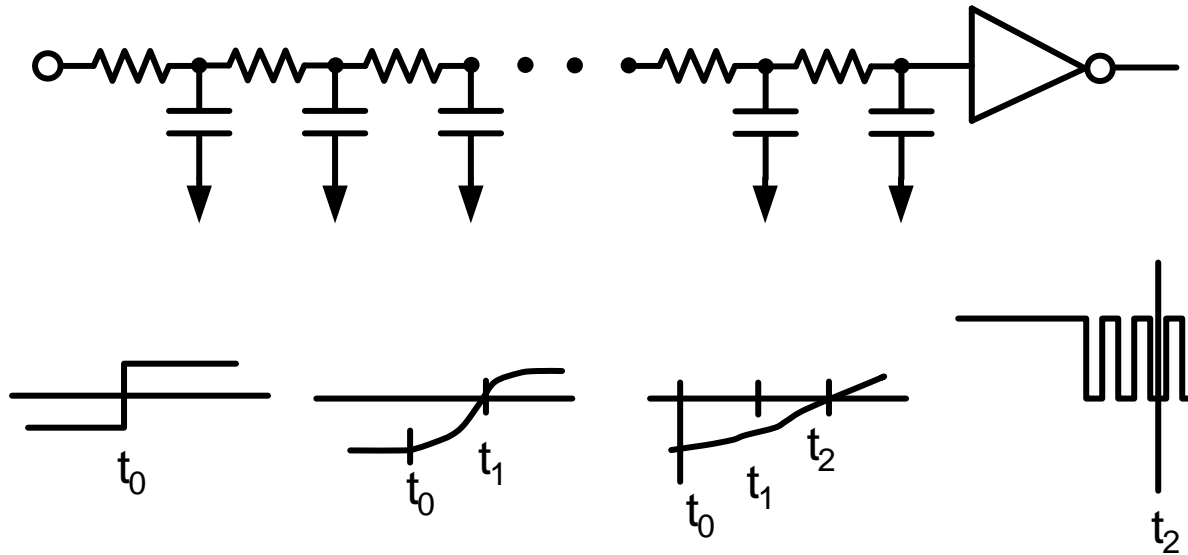
$r$  : resistance per unit length

$c$  : capacitance per unit length

$l$  : length of the wire

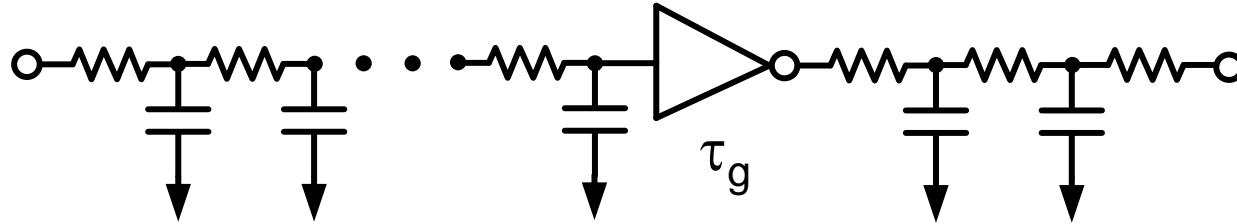
# Distributed RC Effects (cont.)

- Disadvantages of long wire:
  - ◆ Long delay
  - ◆ Reduction in sensitivity to noise



# Distributed RC Effects (cont.)

- Method to improve disadvantages mentioned previously



$$\text{delay} \approx \frac{rc}{2} \left( \frac{1}{2} \right)^2 + \tau_g + \frac{rc}{2} \left( \frac{1}{2} \right)^2$$
$$\approx \frac{rc l^2}{4} + \tau_g$$

If  $\left( \frac{rc l^2}{4} \right) < \tau_g$ , delay time is reduced

If  $\left( \frac{rc l^2}{2} \right) \gg \tau_g$ , more buffers should be used

In actual design, if possible,

$$\frac{rc l^2}{2} \ll \tau_g \Rightarrow l \ll \sqrt{\frac{2\tau_g}{rc}}$$

# Distributed RC Effects (cont.)

- Transmission line effect is particularly severe in poly wire because of the relatively high resistance of this layer. Gate poly layer is the worst one because of its high capacitance to substrate.
- Strategies
  - ◆ Use metal line : small  $r$
  - ◆ Use wider metal for signal distribution line
    - (e.g. clock distribution line) : small  $r$ , a tiny bit large  $C$



# Inductance

- On-chip inductance are normally small.
- Bond-wire inductance is larger.
- Inductance of bonding wires and the pins on packages

$$L = \frac{\mu}{2\pi} \ln\left(\frac{4h}{d}\right) \text{H/cm}$$

$\mu$  : the magnetic permeability of the wire

(typically  $1.257 \times 10^{-8} \text{H/cm}$ )

$h$  : the height above the groundplane

$d$  : the diameter of the wire

- Inductance of on-chip wires

$$L = \frac{\mu}{2\pi} \ln\left(\frac{8h}{w} + \frac{w}{4h}\right) \text{H/cm}$$

$w$  : conductor width

$h$  : the height above the substrate